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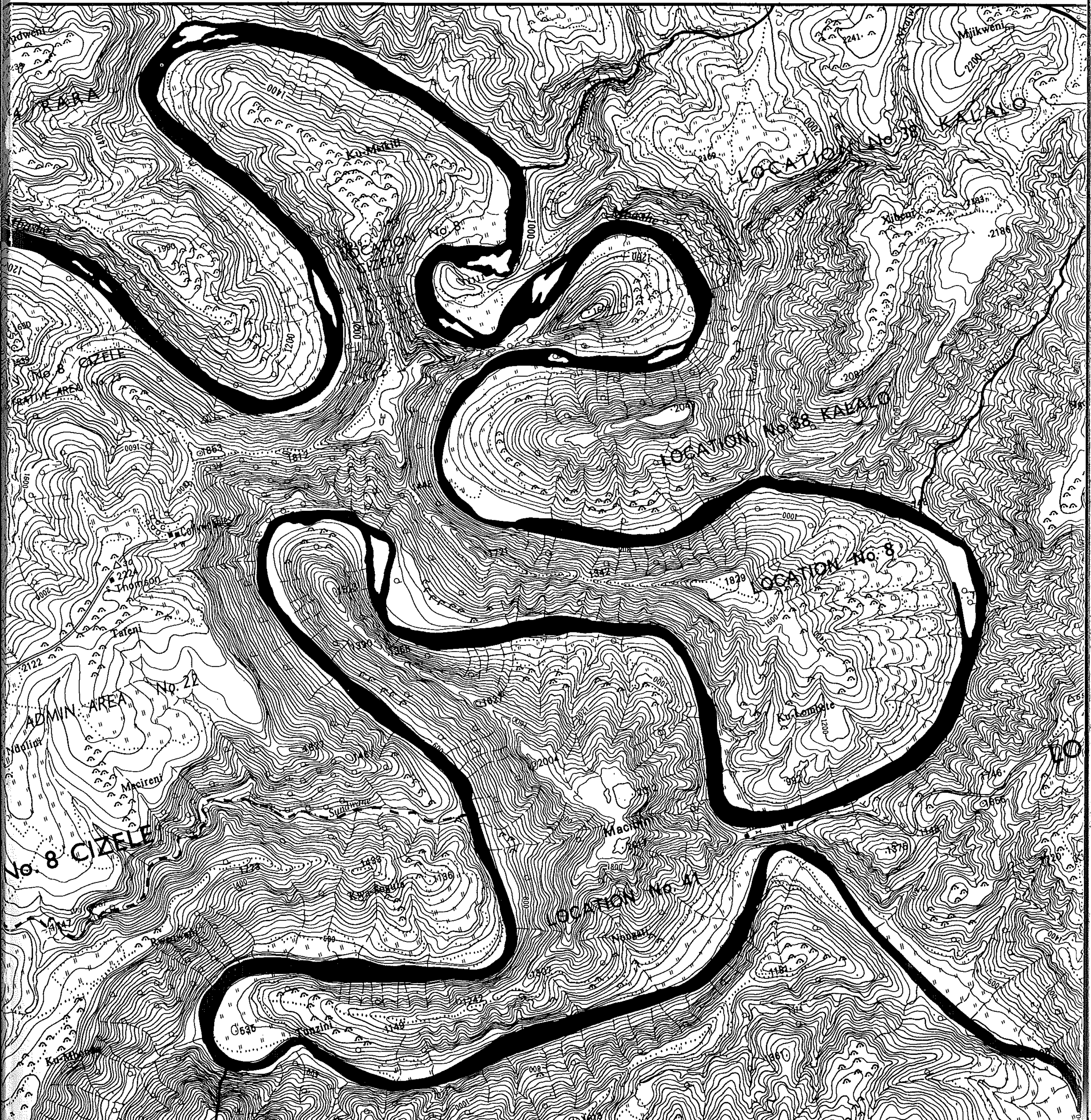
Notes on

Some unsolved problems in river flow

WJR Alexander



DEPARTMENT OF WATER AFFAIRS
Branch of Scientific Services

Technical Report No TR 89

Notes on SOME UNSOLVED PROBLEMS IN RIVER FLOW

by W J R Alexander
February, 1979

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0001

(Cover : Incised meanders in the Mbashe River, Transkei)

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FOREWORD

These notes were prepared for an illustrated presentation to the SIGMA association, which is a group of young engineers in the Department of Water Affairs.

The title of the presentation was to have been "Why can't a river flow in a straight line ?", but it did not take me long to realise that I would not be able to provide the answer to the question other than to demonstrate that this is the most unlikely course that a river will follow. The title of the talk was accordingly changed to "The W-W Bird and some unsolved problems in river flow".

Despite their lack of polish I have reproduced the notes in our Technical Report series in the belief that this informal and somewhat speculative view on the subject may be of wider interest - particularly to those in other disciplines who have some interest in river behaviour.

I have deliberately reduced the mathematical treatment to a minimum and simplified some of the concepts. I have listed the authors, journals and publications that I consulted in the references.

I must record my appreciation of the many invigorating discussions on this subject that I have had over the years with my colleagues in and out of the Department of Water Affairs. I do not name them for fear that they may wish to dissociate themselves from my views.

W J R ALEXANDER
Pretoria
February, 1979

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Notes on

SOME UNSOLVED PROBLEMS IN RIVER FLOW

by W J R Alexander

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1. A FEW QUOTES BY WAY OF INTRODUCTION

1.1 "When you try to explain the behaviour of water, remember to demonstrate the experiment first and the cause next."

Leonardo da Vinci (c 1500)

1.2 "The science of conjecture, or the stochastic science, is defined as the art of estimating to the best as one can, the probability of things so that in our judging and acting we may always choose or follow the best, the safest, the surest, or the most soul-searching way: in this alone rests all the wisdom of a philosopher and all the prudence of a politician."

JACOB BERNOULLI (1713)

1.3 "More than a half century ago, when I was about 10 years old, I found great satisfaction in solving applied mathematical problems of the kind fitting a normal 10 year old boy. One day I pestered my father again with the request for a 'calculation problem'. He just ended a period of work on Braunian movement, determination of the size of molecules and the beginnings of quantum theory, all those things that had absolutely no meaning for me then. So he took a long look at me as if he was not quite sure if he should dare to do what went through his head and finally decided to give it a try. He asked me, 'How long will it take till the ground is wet if it rains at the rate of 10 mm per hour? Just take your time in finding out.' It was the last sentence that got me worried -- it meant that the problem was probably much more involved than just a simple division or some other simple operation. Besides, no matter how I tried to get started always there seemed to be some of the most important information missing.

After doodling for a while but not getting anywhere, I admitted defeat and asked for help. That was naturally just what my father had been waiting for. He asked me first : ' How does rain fall?' 'In drops,' I answered. 'That is very important as you will see. How big is such a drop is really not important that we know exactly.' I ventured 'two millimeters.' 'Good. And how big is the splash from such a drop?' I guessed again: 'a 10 millimeter circle. Now I know how to go about it.' 'Good,' he said and puffed on his cigar seemingly agreeing with me. I went through the process of finding out how many 10 mm circles made a square meter and how long it would take for these drops to fall at the given rain and proudly showed him the result, just to be told: 'That is all very nice, but it is not what I asked you. I asked you how long it takes to wet the area, not how long it takes a given amount of rain to fall. You see, if you take the instant when half the area is wet, where will the next drop fall? Are you sure it will hit a dry spot and wet another 1-cm circle?' I had to admit I did not know. That was the moment where I learned what the significance of probability is and that it takes actually many times longer to wet the area than I had calculated, because many drops are actually ineffective by hitting spots which were already wet. And from the fact that I still remember the entire episode proves that my father's experiment was successful in teaching me at the age of 10 the concept of probability."

HANS ALBERT EINSTEIN (1971)

1.4 "Turbulence was, and still is, one of the great unsolved mysteries of science and it intrigued some of the best scientific minds of the day."

T L VON KÁRMÁN (1967)

1.5 "Directly connected with some of the aspects of the sediment problem is that of turbulence which is infinitely more difficult to attack. ---- If I may again quote my father, he described it as 'too complicated for him'.---- Let me ask only a simple question which may demonstrate our ignorance : Has anybody ever been able to give actual instantaneous velocity distribution (or possibly the pattern of the vorticity lines) in a significant part of a turbulent flow? I do not think so. I do not think that we will be able to understand turbulence unless we are able to give such a flow picture."

HANS ALBERT EINSTEIN (1971)

1.6 "The way in which the passage from a microscale to a macroscale in time and space transforms a stochastic process to a deterministic process or vice versa is particularly emphasized --- because replacing an hydraulic stochastic process by deterministic relationships of averages leads to a loss of information, misleading results, and incorrect problem solutions."

VUJICA YEVJEVICH (1971)

1.7 "Although the first comprehensive treatment of suspended transport as a stochastic process -- was published nearly 25 years ago, it is only within the last five years that the subject has begun to come of age ---. The more recent models have been either random walk types, or deterministic types wherein the random walk is superimposed on a basically deterministic model. The first of these approaches may be likened to a drunkard attempting to reach his destination on foot, and the second to a drunkard attempting to reach his destination by bus."

WILLIAM A SAYRE (1971)

1.8 "When a canal is excavated in fine bed material, and/or a sediment load is admitted at the canal headworks, the designer's problem becomes the highly critical one of choosing a canal design which will pass the required sediment load without undue deposition or scour; he has little margin for error on either side.

It has already been pointed out that existing bed-load formulas have a number of uncertainties even when the effect of side slopes is neglected. Introducing the effect of side slopes or bank competence makes the problem one degree more difficult, and indeed puts it beyond the scope of present knowledge."

F M HENDERSON (1966)

2. WHAT IS THE DIFFICULTY ?

2.1 A problem in four dimensions

River flow is a process which takes place in all four dimensions (three space dimensions and one time dimension). It is very difficult for the human mind to 'picture' four dimensional flow. Mathematicians and engineers tend to resolve the flow into two dimensions at a time eg. into the x–y plane (space) or by using the Langrangian method in problems with time dependence (ie. using the moving water as a point of reference instead of a point on the bank of the channel as in the conventional Eulerian method). Significant errors are often introduced in these simplifications regarding the components in the remaining directions.

2.2 Over simplification

Complex flow is often further simplified by using average values (for example in the Manning and Bernoulli equations). Average values can contain large errors if the relationships are not linear (eg. where kinetic energy is proportional to the square of the velocity).

2.3 Scale dependence

Investigators often overlook the fact that the processes are also scale dependent. What is valid on a macro scale (time or space), may not be valid on a micro scale and vice versa. The concept of streamlines is useful for visualising the paths that parcels of water will follow but is not valid when applied to individual particles of water. A river may adjust its meander pattern over a long period of time but cannot do so in a single flood.

2.4 Cause and effect

Investigators frequently fail to distinguish between cause and effect, and between action and reaction. An example is the often quoted statement that meanders are the form in which the river does the least work in turning and therefore this is the most probable course that it will take. Attempts at determining the resistance offered to flow by dunes in the river bed have been unsuccessful largely because the dunes are a consequence of the energy dissipation phenomena and not the cause.

2.5 Functional relationships

Standard text books on hydraulics unwittingly condition the student's mind to make the assumption that open channel flow is a continuous process that can be described mathematically by continuous functional relationships. In later life the engineer has difficulty in appreciating that these mathematical relationships cannot be used to predict the movement of a particle of water or of an individual grain of sand. These can only be described in probabilistic terms. Hydrologists have long been exposed to stochastic processes, but stochastic hydraulics only started coming into its own in the 1960's, and does not seem to have made much headway since then.

To summarise, the solution of complex river flow problems requires :-

(i) Observation and measurement.
(ii) Visualisation of the process.
(iii) Development of a conceptual model.
(iv) Conversion into mathematical relationships (continuous or stochastic depending on the scale).
(v) Programming the computer to solve the problem.

The second step seems to be the one that is most difficult.

In the pages that follow I will attempt to provide some guidance on how problems in river flow can best be visualised.

En route I will attempt to explain why it is that there is as yet no completely satisfactory answer to a problem which challenged the human mind long before the time of Leonardo da Vinci :-

"Why doesn't a river flow in a straight line ?"

3. SOME DEFINITIONS AND CONCEPTS

3.1 Stochastic processes

The flow in the Vaal River tomorrow will depend very much on the flow measured today, but the flow 365 days from now will be independent of the flow today. In the absence of other information, my best estimate of tomorrow's flow will be that it will be the same as today's flow. My best estimate of the flow 365 days from now will be the average flow (or better still the median flow) for this time of the year.

A stochastic process has three components :-

(i) A deterministic component which can be determined exactly once the initial conditions are known.
(ii) A probabilistic component which cannot be calculated precisely, but can be described in statistical terms.
(iii) Time dependence

In the case of the flow in the Vaal River, the probabilistic component will be zero when estimating tomorrow's flow, but the deterministic component will be zero when estimating the flow a year from now.

3.2 Energy pathways

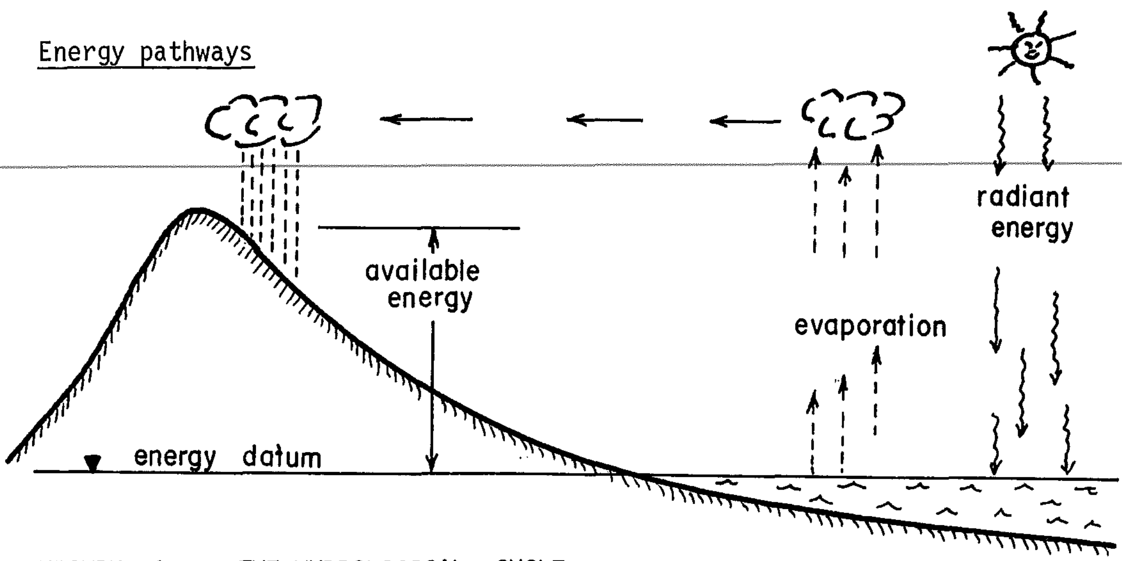


FIGURE 1 : THE HYDROLOGICAL CYCLE.

Incoming radiant energy from the sun drives the hydrological cycle and delivers water to the river catchment. The difference in elevation between the surface on which the rain falls and sea level is the amount of energy that a given mass of water will release to the environment on its journey back to the sea. This is called *potential energy (PE)* and is measured in units of length. Note that this amount of energy is independent of the horizontal distance travelled or time taken to reach the sea. It is therefore independent of velocity and channel slope.

Potential energy is converted to other forms of energy en route to the ocean, and is ultimately dissipated to the environment as heat energy.

Potential energy is converted to *kinetic energy* when flow is initiated.

Translational kinetic energy is the energy stored in a mass of water moving forward in the general direction of flow. There is a free exchange between potential and translational energy as a mass of water accelerates and decelerates in a forward direction, although each transformation is accompanied by some energy loss, which can be large in the case of diverging sections for example.

Rotational kinetic energy is the energy stored in rotating water such as takes place in heliciodal flow, vortices and turbulence. Nearly all the rotational energy is ultimately transformed into heat energy as it cannot readily be transformed into translational or potential energy.

When the terms 'energy loss' or 'energy consumption' are used the implication is that the energy is irretrievably lost from the water system.

3.3 Other properties of water

Viscosity is the property of water which resists deformation and is due to the cohesiveness of the water molecules. It is proportional to the velocity gradient at that point.

Shear stress (τ) has the units N/m^2 and when multiplied by the area on which it acts equals the *shear force* applied to that area.

Mass density of the fluid (ρ) is the specific weight divided by the acceleration of gravity (g).

Specific weight (γ) is in units of weight per unit volume $= \rho g$

3.4 Action and reaction

I move my body forward by exerting a force on the ground. *If the ground doesn't move* it offers full reaction to the applied force. There is therefore only one possible result - my body must move forward. This process only has one degree of freedom.

If I tread on a banana peel the result will be quite different. In this case there will not be sufficient reaction to the applied force and therefore there can be no forward movement of my body.

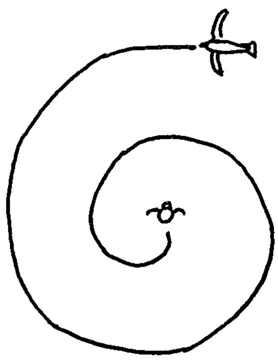
Similarly the outer bank of a bend in the river does not force the stream to change its direction of flow. It resists the forward movement of the water.

The significance of these statements will become apparent later.

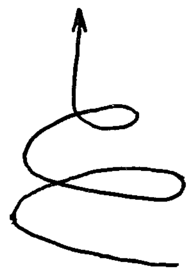
3.5 The W-W Bird - a problem of visualisation

"When pursued by an enemy, the W-W Bird flies round in ever diminishing circles ....".

The person who observed the incident erred in his interpretation because he only had a two-dimensional view and therefore failed to perceive that the bird was also moving in the third dimension which was away from the viewer. The bird reduced its circular velocity but simultaneously increased its vertical velocity. Its orientation changed as a result of its change in flight direction. It appeared to get smaller when in fact it was climbing up into the sky. The observations were sound - it was the interpretation that was at fault.



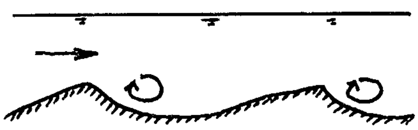
View from below



Side view

FIGURE 2 : THE FLIGHT PATH OF THE W-W BIRD.

There is an analogy in river flow. Beware of diagrams showing two-dimensional flow paths which look like this :-



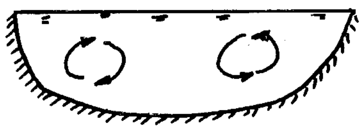


FIGURE 3 : HELICOIDAL FLOW.

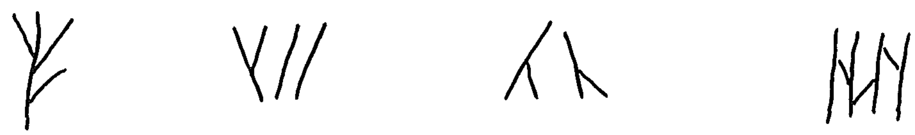
Water cannot rotate in one plane without a component in a direction perpendicular to the plane of rotation. Watch the phenomenon next time you remove the bath plug - and remember the lesson of the W-W bird.

4. SOME OBSERVATIONS

4.1 Classification of river channels

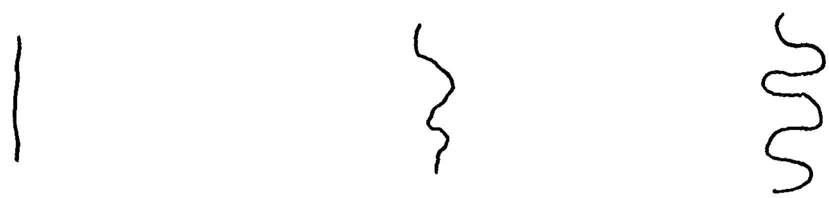
River channels can be divided into multiple and single-thread watercourses :-

Multiple thread :-



stem converging diverging braided

Single thread :-



straight irregular meandering

FIGURE 4 : TYPES OF RIVER CHANNELS

All of these can be observed in nature, but long straight channels are very rare and seldom have lengths greater than ten times the channel width.

Where straight reaches extend for long lengths, these are usually due to geological controls eg. the Vaal/Orange system of the Northern Cape, and the gorges below Victoria Falls.

Where no geological controls are present, for example on alluvial plains, rivers do not flow in a straight, single thread channel. In fact the flatter the valley slope the more sinuous the river course.

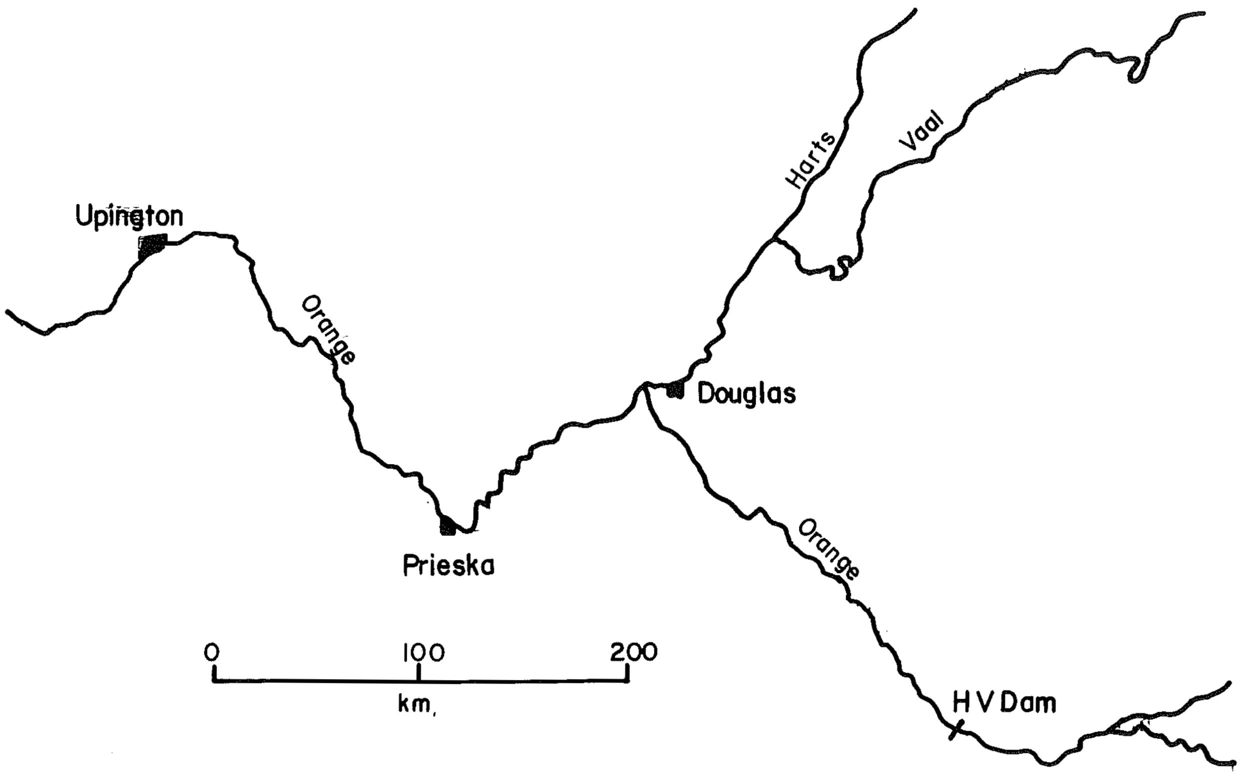


FIGURE 6 : MARKED LINEATION OF THE ORANGE AND VAAL RIVERS AT THEIR CONFLUENCE. THE CAUSE IS NOT KNOWN

In some cases uplift has preserved the original meandering pattern which has become deeply incised into the underlying rock although the flow and sediment carrying capacity of the present river may differ greatly from the original condition. A good example is Mbashe River in the Transkei. Other notable examples are the Grand Canyon of America and the Fish River Canyon in South-West Africa.

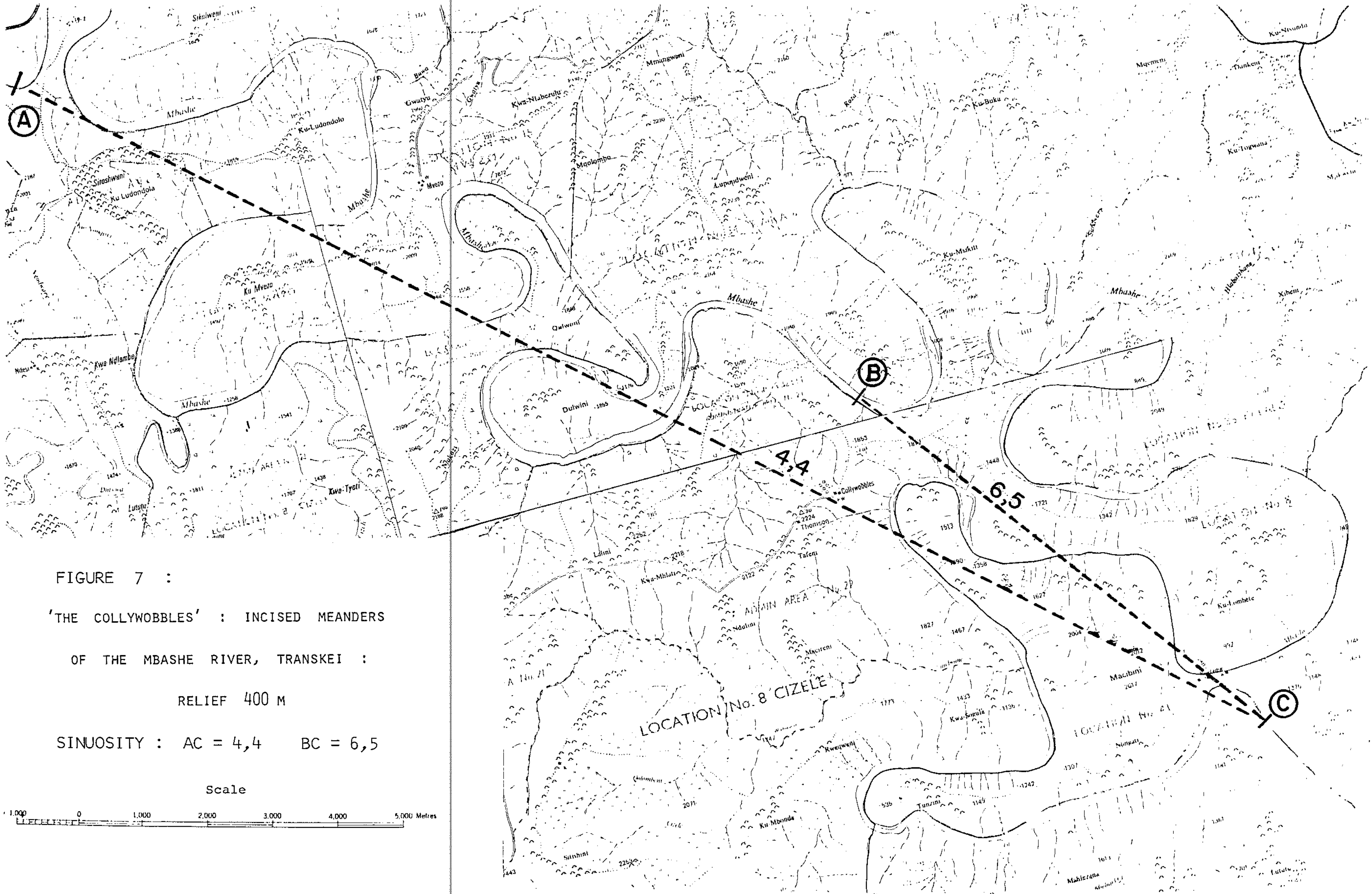


FIGURE 7 :

'THE COLLYWOBBLES' : INCISED MEANDERS OF THE MBASHE RIVER, TRANSKEI :

RELIEF 400 M

SINUOSITY : AC = 4,4 BC = 6,5

Scale

1,000 0 1,000 2,000 3,000 4,000 5,000 Metres

5. SOME HYDRAULIC LAWS AND THEIR APPLICATION TO RIVER FLOW

5.1 Case one : a uniform channel with no resistance to flow

Consider a constant rate of flow along a straight, uniform channel which has a smooth perimeter which does not offer any resistance to flow.

Two basic laws are applicable.

Newton's second law states that a mass will accelerate linearly when a force is applied to it.

Force = Mass x Acceleration

$$F = M \cdot A$$

The force applied to the water is the force of gravity. In the absence of a resistance to this force, the water will continue accelerating down the channel.

From the *law of conservation of mass* for conditions of steady flow the rate of flow out of the section will be equal to the rate of flow into the section. The rate of flow is equal to the velocity multiplied by the cross-sectional area. As the water accelerates the velocity increases and the cross-sectional area must therefore decrease proportionately.

$$Q = A_1 . V_1 = A_2 . V_2 = \text{constant}$$

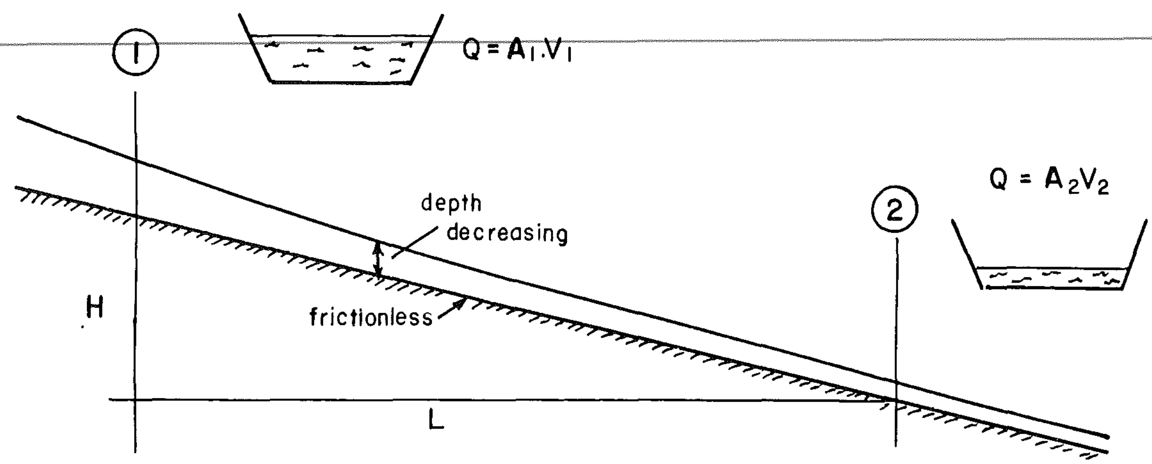


FIGURE 8 : STEADY FLOW ALONG A STRAIGHT, UNIFORM, FRICTIONLESS CHANNEL.

5.2 Case two : a rough uniform channel

Consider the same conditions as in case one, but this time the channel has a rough boundary ie. it offers a resistance to flow.

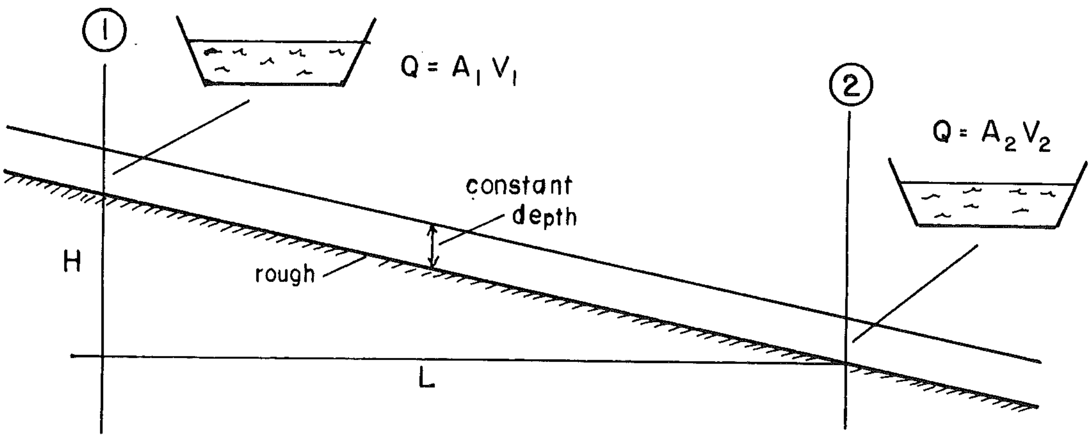


FIGURE 9 : STEADY FLOW ALONG A STRAIGHT, UNIFORM CHANNEL WITH A ROUGH PERIMETER.

Because of the resistance to flow, the water velocity will reach a constant equilibrium value when the rate of energy available to maintain forward flow equals the rate of energy consumed in overcoming the resistance to forward movement (action and reaction).

Several formulae are available for determining what the equilibrium velocity will be. The Manning formula is the best known :-

$$\text{Velocity} = \frac{(\text{hydraulic radius})^{2/3} \times (\text{slope})^{\frac{1}{2}}}{\text{roughness coefficient}}$$

$$v = \frac{1}{n} \cdot r^{2/3} \cdot s^{\frac{1}{2}}$$

where the hydraulic radius is defined as the wetted perimeter divided by the cross-sectional area. This is approximately equal to the depth of flow for a wide, shallow channel.

The unstated assumption in this formula is that the velocity is the <u>*mean*</u> *velocity at that section of the river.* The dependant variable should therefore be denoted by $\bar{v}$ and not v.

5.3 <u>What paths will streamlines follow</u> ?

The velocity of the water at a cross section of a river can be measured by current meters. The following are two-dimensional representations of the velocity field.

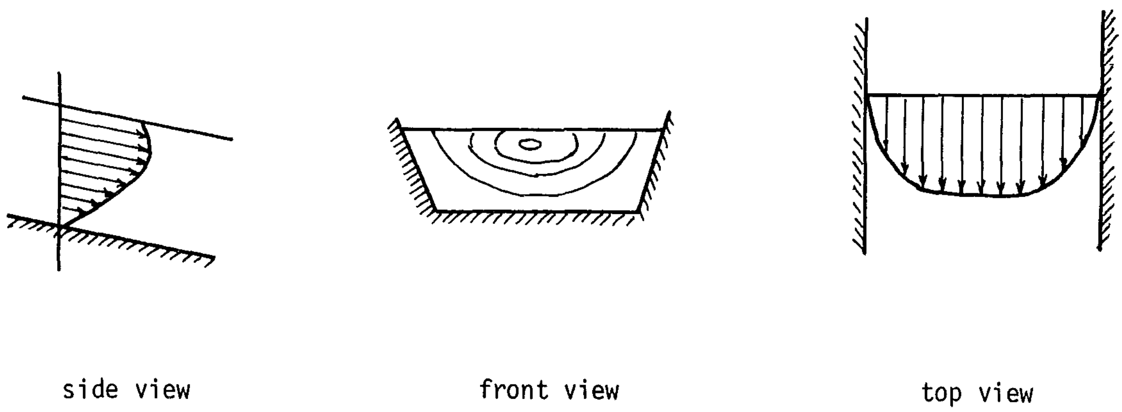


FIGURE 10 : TYPICAL VELOCITY FIELD IN A UNIFORM TRAPEZOIDAL CHANNEL.

Note the depressed zone of maximum velocity. Why is this not at the water surface ?

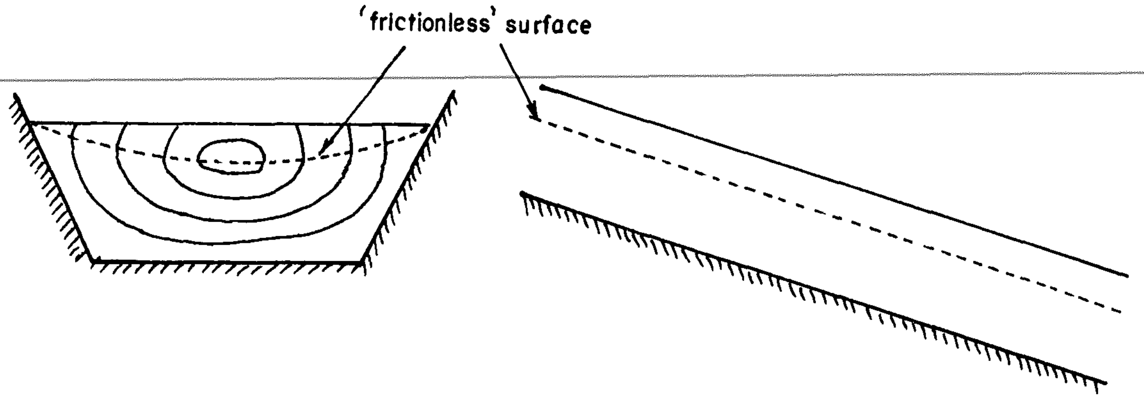


FIGURE 11 : HYPOTHETICAL FRICTIONLESS SURFACE

A line can be drawn through the edges of the free water surface and the point of maximum velocity such that it crosses the isovelocity lines at right angles. Therefore there cannot be any difference in velocity between any pair of points immediately above and below this line. (ie. no velocity gradient across the line).

Therefore there can be no resistance to flow across the line to balance the force of gravity resolved down the slope.

As there is no resistance to flow across this surface, the water above it must accelerate - but it doesn't!

Henderson (1966) states : "This paradox can only be resolved by postulating secondary flows in the directions shown by the arrows"

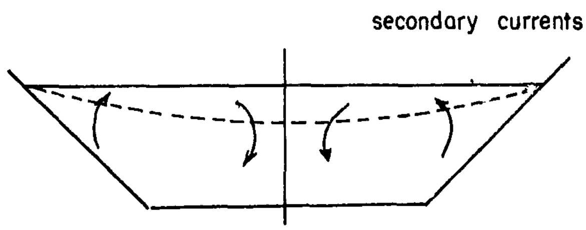


FIGURE 12 : POSTULATED SECONDARY FLOWS

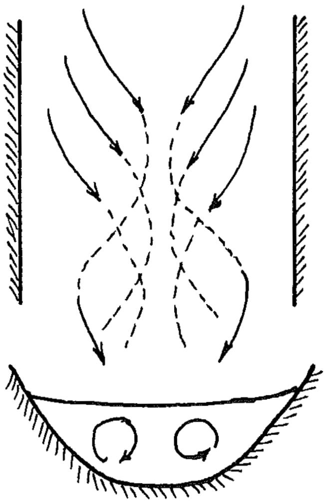
Henderson then continues ".... It must be realised that none of this argument explains why the secondary flows occur, and in fact no complete explanation has yet been found for the occurrence of these flows in straight channels."

If we had been present during the measurement of the water velocity we would have noted that the current meter is such that it is free to orientate itself in both the horizontal and vertical directions, but that this orientation was not taken into account in the measurements. Had this been done it would have been observed that the velocity being measured was not perpendicular to the cross-section, but was at a slight horizontal and vertical angle to it, and that these angles varied from place to place in the section.

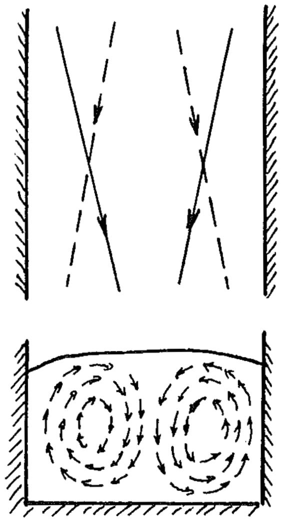
Note also the use by Henderson and most other authors of the term 'secondary flow' . This is confusing. The term 'helicoidal flow' is conceptually more correct and assists in visualising the flow pattern.

5.4 Helicoidal flow

Helicoidal flow was recognised independently by two investigators as long ago as 1882 and rediscovered in 1909 !



Max Möller (Germany, 1882)

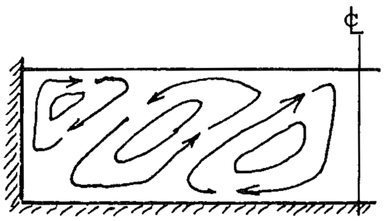


F P Stearns (USA 1882)
A H Gibson (USA 1909)

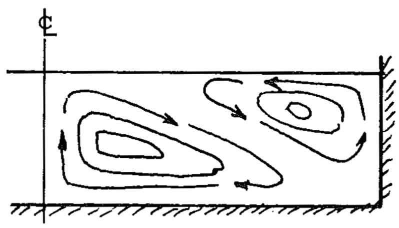
FIGURE 13 : VELOCITY FIELDS POSTULATED BY MÖLLER, STEARNS AND GIBSON.

Möller postulated that the water surface should be concave upwards whereas Gibson found that it was convex upwards. "... because roughness protuberences, like any other obstacles, tend to produce a locally lowered pressure" !!

Other suggestions have been :



Prandtl (Germany, 1927)



Powell (USA 1946)

FIGURE 14 : VELOCITY FIELDS POSTULATED BY PRANDTL AND POWELL.

The figure below shows the measured flow velocities and expected circulation pattern in the Po River in Italy.

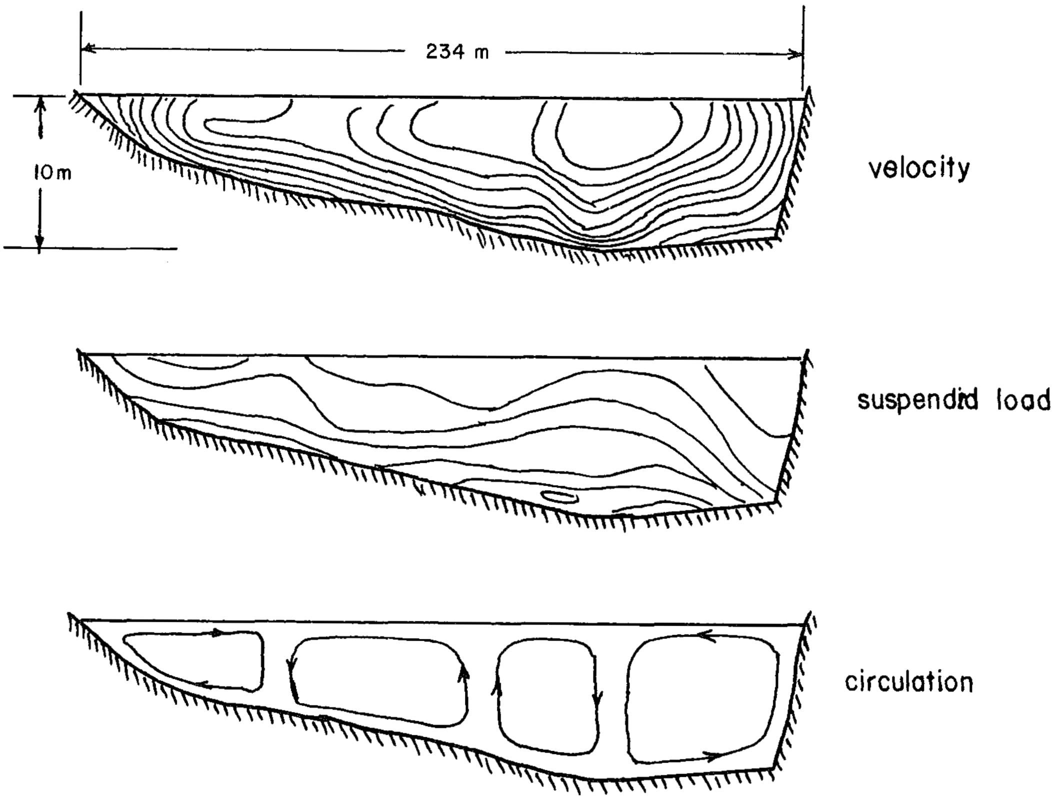


FIGURE 15 : VELOCITY FIELDS IN THE PO RIVER. (After Vanoni, 1946)

All the above configurations may exist in nature, depending on the width/depth/flow relationships of the channel.

We can now visualise channel flow on two scales :-

Macro scale : All streamlines are assumed to be parallel to one another and follow the direction of the river channel. Average velocities are used in the calculation methods.

Meso scale : Streamlines are assumed to follow helicoidal paths within two or more cells. A parcel of water (but not particle as we will see shortly !) starting at the river bank will accelerate downhill and move towards the centre of the stream, dip below the surface where it reaches its maximum velocity, continue on an angled path towards the bed of the river decelerating on the way, flow along the river bed where the maximum deceleration takes place, rise up and along the bank of the river until it reaches the water surface where it starts on the next acceleration-deceleration cycle.

In this visualisation, we can now conceive how the velocity of the flowing water is restrained by the perimenter of the river channel, and conversely how the flowing water acts on the perimeter and scours out any weak spots thus initiating departures from a straight channel.

But there are still more conceptual problems ahead ! These relate to flow on a *micro scale* and involve the as yet unresolved quantification of turbulent flow.

6. BERNOULLI AND THE LAW OF CONSERVATION OF ENERGY

6.1 Bernoulli equation for steady flow

Standing water in a river catchment has an energy potential equal to the height of the surface of the water above sea level.

When water commences flowing down a slope some of the potential energy is converted into kinetic energy which is proportional to the square of the velocity.

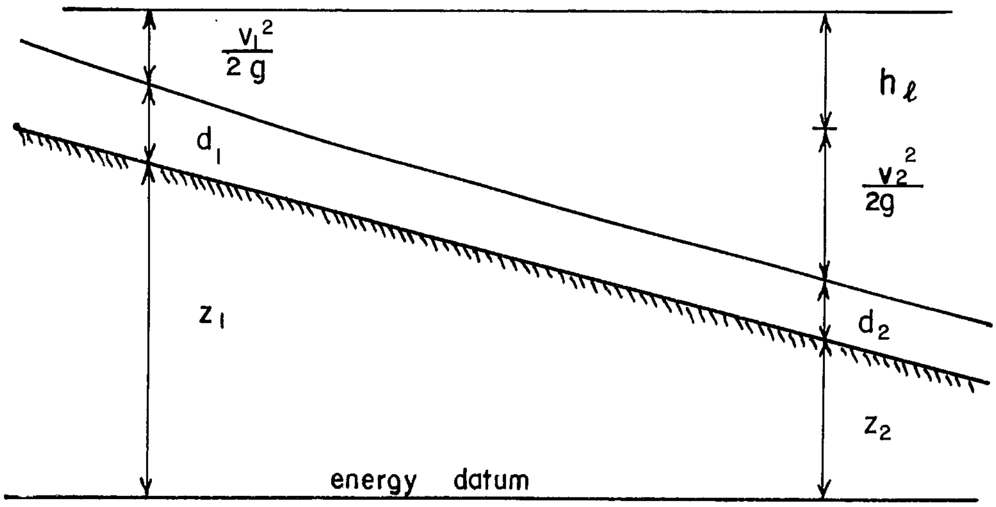


FIGURE 16 : VARIABLES IN THE BERNOULLI EQUATION.

The simplified form of the Bernoulli equation is :-

$$z_1 + d_1 + \frac{v_1^2}{2g} = z_2 + d_2 + \frac{v_2^2}{2g} + h_\ell$$

where h_ℓ is the energy lost to the environment in the section 1 - 2.

Note that all velocities are *mean* velocities at the cross-section. This leads to the next complication :

FIGURE 17 : ASSUMED AND ACTUAL VELOCITY PROFILES.

The mean of the sum of the squares of the actual velocities is not the same value as the square of the mean velocity !

This is another macro/meso scale inconsistency which can usually be ignored in most hydraulic problems, but leads to difficulty when dealing with streamlines. Consider the following.

6.2 Rotational energy

Where conditions are such that parcels of water rotate in addition to their forward movement, potential energy is converted into kinetic energy in the process.

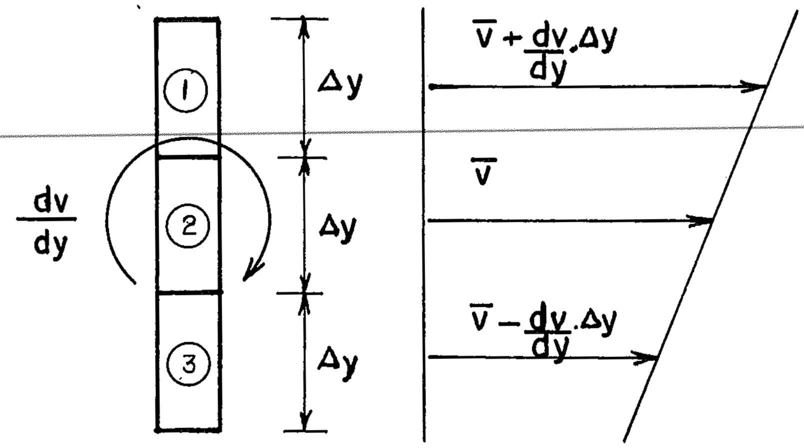


FIGURE 18 : ROTATIONAL FLOW (after Rooseboom).

The term $\frac{dv}{dy}$ is the rate of change of velocity in the vertical direction. The vertical column of water must therefore move forward at a mean velocity $\overline{v}$ and *rotate with mean angular velocity* $\frac{dv}{dy}$.

The kinetic energy of the column therfore consists of two parts, viz

(i) Translational energy in the direction of flow which is a function of forward velocity.

(ii) Rotational energy which is a function of both the velocity gradient across the profile as well as the moments of inertia of units which rotate as such.

Note that there is also a velocity gradient in the lateral direction causing horizontal rotation.

The total energy in the section can be defined as :-

total energy = translational energy + rotational energy

ie. $$\alpha \cdot \frac{\overline{v}^2}{2g} = \frac{\overline{v}^2}{2g} + (\alpha - 1) \cdot \frac{\overline{v}^2}{2g}$$

Rooseboom ascribes the phenomenon of convex curvature of the water surface of a river observed by Alexander (and Gibson seventy years before him !) as follows :-

> "In river flows, flow velocities along flood plains are normally small while numerous eddies are formed due to the presence of large obstacles such as trees. The rotational energy content of such eddies with large amounts of inertia will be high. It is thus possible to explain why the water level in a river above the main channel is often higher tha above the flood plains (a difference of ± 0,3m has been observed in the Orange River at Upington)."

Rooseboom shows that α values expressed in terms of the mean channel velocity will normally be in the range 1,03 to 1,12 and suggests using an average factor of 1,05. Henderson quotes α-values of 2 and more in natural rivers in terms of average river velocity including overbank flow.

Note that unlike translational energy, rotational energy cannot readily be transformed back to potential energy and is therefore permanently lost to the system.

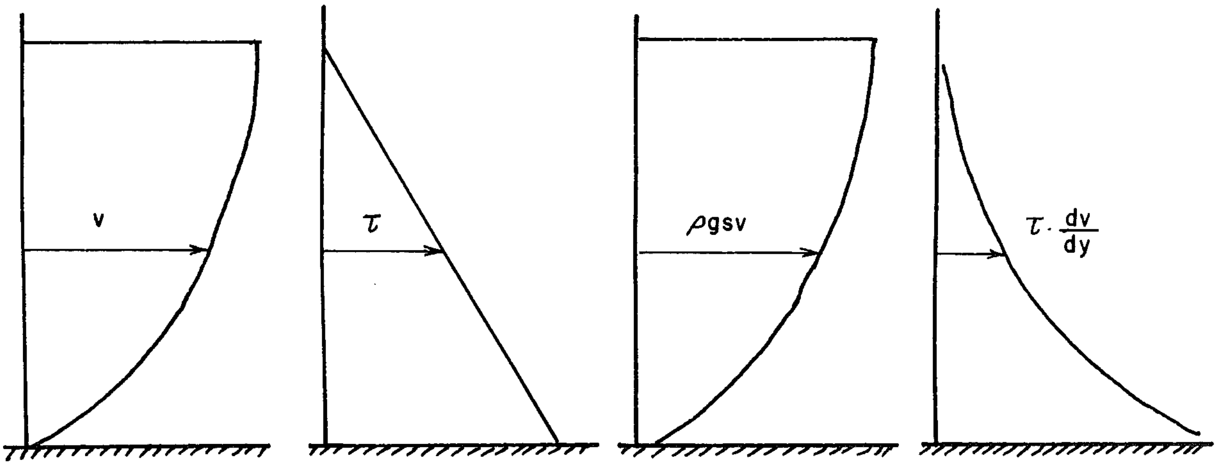
6.3 Rotation or shear ?

In my view the argument in the paragraph above only holds good when the water actually rotates, and is not valid when the water shears instead of rotating. I will expand on this when discussing resistance to flow, but in the meantime I would like to state that *where a velocity gradient exists, a parcel of water will either shear or rotate depending on the condition along its boundaries.*

7. RESISTANCE TO FLOW

7.1 Channel roughness

The roughness of the channel perimeter resists the forward movement of the water, and this resistance is transmitted through the water body. It may by visualised as follows :



(a) velocity profile

(b) Shear stress profile

(c) Power applied in maintaining flow

(d) Power consumed in resisting flow

FIGURE 19 : VELOCITY, SHEAR STRESS AND POWER PROFILES PER UNIT VOLUME OF AN INFINITELY WIDE STREAM (after Rooseboom).

The layer immediately in contact with the channel perimeter is restrained from moving due to the rigidity and roughness of the material constituting the perimeter. The viscosity of the water restrains but does not prevent the forward movement of the next layer. The actual velocity of subsequent layers will therefore depend on the magnitude of the force applied, the velocity of the water in adjacent layers, and the viscosity of the water. The resistance of the bottom layers is transmitted throughout the profile in this way.

The shear stress (units of force per unit area) decreases linearly from the bed to the water surface.

The power applied in maintaining flow is directly proportional to velocity and therefore has the same profile.

Rooseboom showed that the mean velocity of flow could be calculated as a function of the eddies along the flow boundary which in turn are a function of boundary roughness, as in the modified Chezy equation :-

$$\bar{v} = 18 \left(\log \frac{12R}{k}\right) \sqrt{Rs}$$

where R is the hydraulic radius (also called hydraulic mean depth)
s is the slope
and k is a measure of absolute roughness.

This formula is theoretically sounder than the empirical Manning and Chezy formulae.

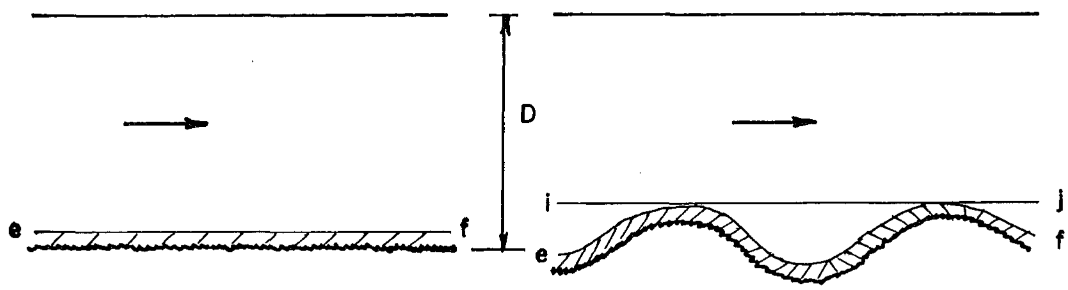
The k factor can be visualised more readily than Manning's n. Here are some examples.

Surface	Absolute roughness $k(m)$
Glass	0,000 2
Concrete - steel form	0,000 6
- unfinished, rough	0,008
Channel - earth	0,01
- uniform gravel	0,08
- straight sections	0,2
Rivers - clean, winding	1
Mountain streams - cobbles and boulders	2

[Note : In a recent hydrology training course, students were shown slides of various types of channel material and asked to estimate the absolute roughness factors. The results were remarkably consistent.]

7.2 Grain roughness vs form roughness

Consider the following two situations



(a) grain roughness (b) form roughness

FIGURE 20 : TYPES OF ROUGHNESS.

In both the above situations, the layer of water below the surface e-f is assumed to be immobile due to the roughness of the solid surface below it. The following layer of water shears across the surface e-f.

However, in Figure 20(b) there is another surface i-j which is shorter than e-f (and therefore smaller area in the horizontal plane), across which shearing could take place, leaving a zone of dead water between the surfaces e-f and i-j. If this were to happen the effective roughness of the channel would be the same regardless of the size of the material in the bed.

In para 6.3 above I postulated that *"where a velocity gradient exists, a parcel of water will either shear or rotate, depending on the conditions along its boundaries."*

I would now like to take this one step further and postulate that *a parcel of water will rotate rather than shear wherever boundary conditions permit this.*

In Figure 20(b), water will therefore rotate within the 'dead water' zone rather than shear across it. The diameter of the rotating cell can be visualised as being the same as the height of the form roughness, which is the k-factor in Rooseboom's equation in paragraph 7.1 above.

A little thought will show that this rotation cannot take place unless the water exits from the centre of rotation in a direction perpendicular to the plane of rotation (the W-W-bird problem again). Also, form roughness does not consist of cylindrical elements but is closer to hemispherical elements.

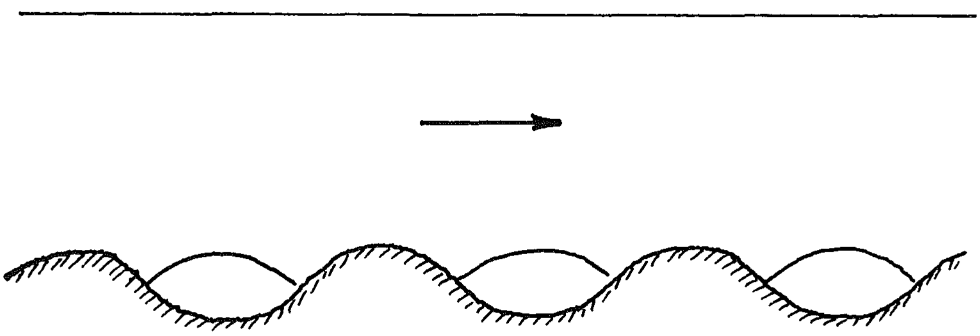


FIGURE 21 : FORM ROUGHNESS APPROXIMATES HEMISPHERICAL ELEMENTS RATHER THAN CYLINDRICAL ELEMENTS.

The turbulent nature of the flow near the bed can be appreciated.

7.3 The alpha-coefficient again

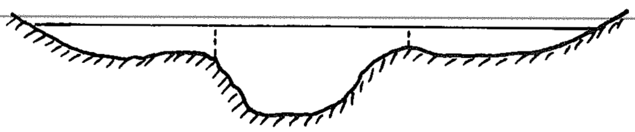
Going back to rotational energy described in para 6.2, it should be possible to relate this to bed roughness as well. Rooseboom showed this to be so, and developed the relationship between the hydraulic radius, absolute roughness and α-coefficient.

This is illustrated in the first two columns of the table below which are from Rooseboom's figures. From these it is possible to determine the ratio of the total available energy that is used in maintaining translational and rotational flow respectively.

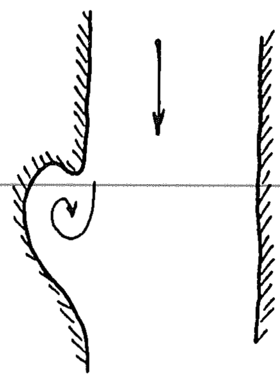
R/k	α	Translational energy	Rotational energy
(2)	(1,25)	80 %	20 %
(5)	(1,16)	86	14
10	1,12	89	11
100	1,06	94	6
1 000	1,03	97	3
10 000	1,02	98	2

From this table it will be seen that if the bed material consists of boulders with dimensions of one tenth of the hydraulic radius, that only 89% of the available energy will be used in maintaining flow in the forward direction, while 11% will be used for maintaining rotational flow due to the protuberences along the channel perimenter.

But this is not the only cause of rotational flow :-



(a) Overbank flow



(b) sudden expansion

FIGURE 22 : OTHER FORMS OF ROTATIONAL FLOW.

When overbank flow occurs there will be a significant difference in the mean velocities of the flow in the channel and on the flood plain, which will cause helicoidal rotation and exchange of flow.

Sudden expansions in the river channel will also cause rotational flow to take place.

Henderson reports that the α-coefficient can be larger than two which implies that under these circumstances less than 50% of the total energy generated in the reach of the river is available for maintaining forward movement of the water. (This is my interpretation - others disagree).

7.4 Visualisation of head loss

We have seen how gravitational forces cause water to move down a slope. In the process the potential energy is reduced as the water loses elevation, but this is compensated by an increase in kinetic energy. The total energy content of a unit volume of water would remain the same as it lost potential energy and gained kinetic energy during its passage down the river were it not for the resistance to flow offered by the river channel.

Energy is 'lost' or 'consumed' in overcoming this resistance. Equilibrium is reached when the flow conditions are such that the energy loss over a unit length of channel equals the loss in elevation over the same length. As we noted before this energy loss per unit length of channel is independent of the channel roughness.

The energy is lost to the system in the form of heat energy, which in turn is caused by viscous shearing within the water body.

Channel roughness therefore determines *'how'* the conversion to heat energy takes place, but does not determine *'how much'* energy is converted.

Later on in these notes I will quote Yang who, in the context of river meanders, submitted the following as a basic law :-

> "During the evolution toward its equilibrium condition, a natural stream chooses its course of flow in such a manner that the rate of potential energy expenditure per unit mass of water along its course is a minimum. This minimum value depends on the external constraints applied to the stream."

I see no reason why we cannot apply the same law to flow paths within the water body. Wherever the external constraints (eg. protuberances in the river bed) permit it, the parcel of water can reduce the rate of potential energy expenditure by increasing its flow path. It can achieve this by helicoidal rotation.

In a relatively smooth channel, helicoidal rotation of individual parcels of water is inhibited and the rate of potential energy per unit length of flow path is greater than in helicoidal flow.

(Note that the rate of expenditure of energy per unit length of channel is the same in both cases).

To summarise, potential energy is converted into two types of kinetic energy viz translational energy and rotational energy. The relative proportions of these two types of kinetic energy depend on channel roughness. An increase in channel roughness does not cause an increase in energy loss, nor does it cause an increase in turbulent or rotational flow. Increase in channel roughness provides an opportunity for the water parcels to rotate naturally and thereby follow a longer path and reduce the rate of energy expenditure per unit length of flowpath.

As we will see later, when a river flows through alluvium it has the opportunity to mould the material in the channel bed and thereby create its own 'roughness'. Only under certain special conditions does the river in alluvium create a plane bed.

A smooth channel bed must be seen as a constraint. When this constraint is removed, the river will develop a rough bed - not a smooth one.

7.5 Instability

As rotational flow can and does take place in any direction, streamlines at a fixed point do not follow a constant direction but change direction - sometimes rapidly - with time. The flow direction is therefore unstable, and the greater the energy expenditure on rotational flow, the greater the instability of the phenomenon. Under these conditions surges occur which give rise to the familiar boils on the water surface.

These surges may be quasi-regular. A good example is shown in Figure 23 where the difference in the smooth trace of the water level during low flow conditions changes to rapid oscillations at higher discharges. In this case the oscillations were damped by the restricted inlet into the recorder well - the amplitude of the changes in water level of the river itself was much larger. The oscillations were caused by rotational flow which was due to a sudden expansion in the river channel downstream.

(The last sentence is badly worded. I should rather have said that the sudden expansion of the river channel removed a constraint on the flow and allowed the water to rotate !)

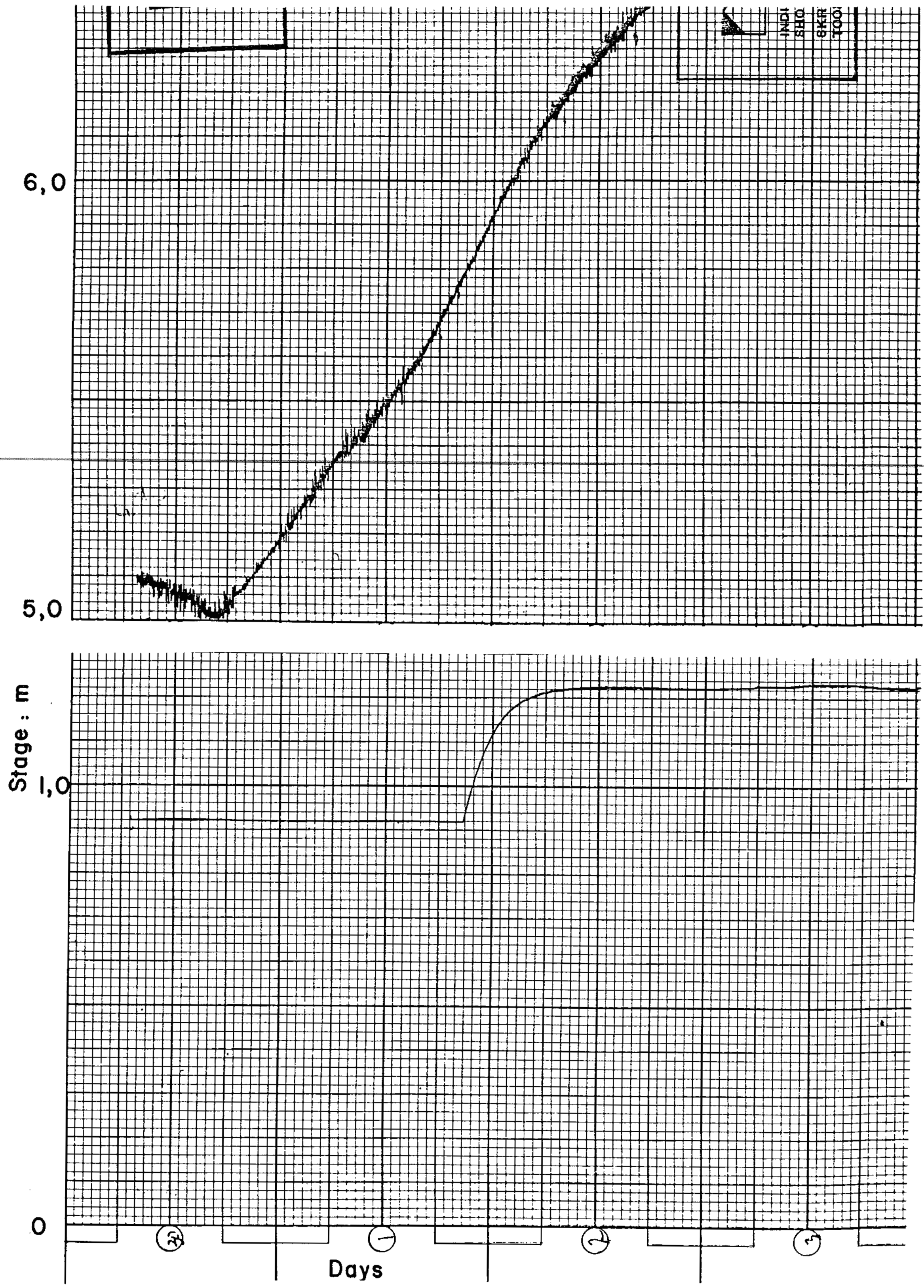


FIGURE 23 : OSCILLATION OF THE WATER LEVEL IN THE ORANGE RIVER AT UPINGTON AT HIGH FLOWS DUE TO SUDDEN EXPANSION OF THE RIVER CHANNEL.

8. SEDIMENT TRANSPORT

8.1 No sediment - no sediment transport !

Much has been written in the past and will no doubt continue to be written in the future on the complex subject of sediment transport. Mathematical relationships between river flow and sediment transport have often been confounded by field observations. What these investigators have overlooked is that the mathematical relationships refer to the river's sediment transport potential. The actual sediment load will also depend on the availability of sediment. If no sediment is available for transport, none will be transported !

This is well illustrated by the variation of sediment discharge with water discharge in the Volga River for two consecutive seasons :

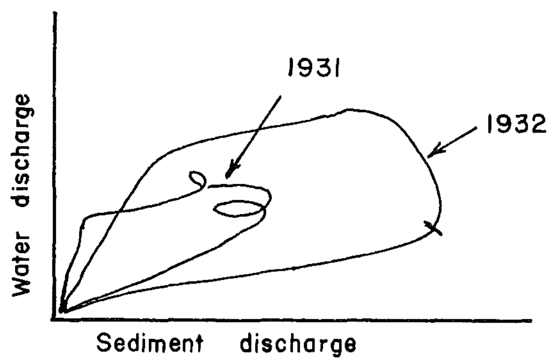


FIGURE 24 : VARIATION OF SEDIMENT DISCHARGE WITH WATER DISCHARGE. AFTER CHEBOTAREV (1966)

Strangely, the Russians have also noted the double peak phenomenon that we have noted in South Africa (see Technical Note 87 and its references) but they attributed it to :-

"... turbidity formed upon erosion of the soil by the surface runoff (vertical hatching) and ... turbidity formed as a result of channel scouring (horizontal hatching). The maximum scouring effect occurred before the maximum water discharge."

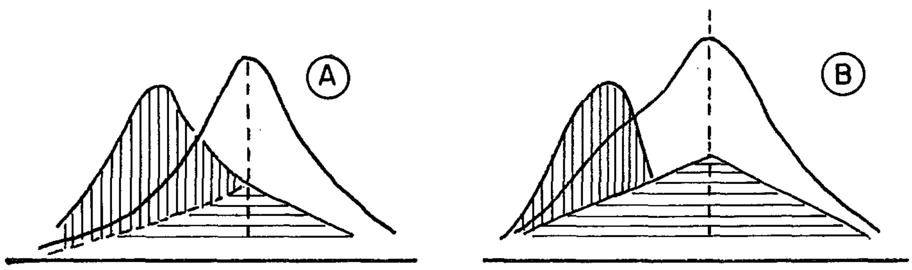


FIGURE 25 : TIME DEPENDENCE OF TURBIDITY AND SEDIMENT DISCHARGE FOR STREAM RISE DURING LOW-WATER YEARS (A) AND DURING HIGH WATER YEARS (B). AFTER CHEBOTAREV (1966)

Chebotarev explained that during a year of high flow channel erosion markedly increases, leading to the formation of the second turbidity peak. Investigations at four sites along the Volga River showed that the coincidence or non-coincidence of the turbidity peaks with the discharge peaks depend on the discharge, water velocity, and type of channel bed. *"Another factor influencing the above characteristics is the amount of erosion occurring in the watershed".* The italics in the above quote are mine. It is our view in South Africa that it is indeed the availability of sediment from the catchment which controls the double-peak phonomenon we have observed (see Technical Report No 87).

8.2 Bed load, suspended load and wash load

Even the nomenclature in sediment transport studies has been a source of differences of opinion. All agree that sediment particles are transported by being maintained in suspension in the flowing water or by being rolled along the bed, either continuously or discontinuously. The arguments centre around whether or not these forms of transport are a single process and can therefore be described by a single mathematical expression or not.

I share the view that these are different modes of transport with different driving forces and therefore they can not be described by a single mathematical function. Many will disagree.

Perhaps the safest categorisation of sediment load is that of bed material load and wash load. The former is the material which constitutes the bed of the river at the point of interest. Looking at the river as a whole, the bed material will become progressively finer in the downstream direction. The primary source of bed material is the abrasion and disintegration of material in the bed upstream. If a river system is in dynamic equilibrium, then over a period of time there will be constant rate of transport of this material through the system from the watershed to the sea. This material will be coarse at the headwaters and finer at the river mouth, but the total load will be the same. The rate of transport will be determined by the rate of disintegration of the bed material and degradation of the river channel.*

Wash load is the material which enters the system via surface erosion of the river basin. The sediment carrying capacity of sheet flow over a surface both volumetrically as well as the maximum grain size, is very much less than that of the combined volume of water once it reaches the drainage channels. With the possible exception of the downstream end of the system, the wash load entering a river will be significantly finer than the bed material. As the bed material is the result of transport to that point, wash load will be readily transported beyond it. It is possible that the coarser particles of the wash load will become part of the bed load at a reach well downstream of the point of entry.*

All mathematical expressions relating to total sediment load are therefore doomed from the start if they are used to estimate the sediment load at a particular point in a river associated with a particular flow, because the total wash load is independent of the flow conditions at that point.

However they can be used and presumably the authors only intended that they should be used, for estimating the maximum potential sediment load

(* See the alternative visualisation of sediment transport in Technical Report No 87) associated with the given conditions, whether or not the transport was through suspension or movement along the bed.

In my view even the latter assumption is tenuous, because as will be seen later, there is a complex, discontinuous relationship between the rate of sediment transport and all the other properties of river flow which are associated with energy losses in the system.

8.3 Colloidal particles

The finest sediment consists of colloidal particles which are held in suspension by forces other than the turbulence of the water. Therefore the concentration of this material in the water is independent of the flow characteristics and cannot be predicted by hydraulic equations. Indeed, the presence of this material in the water adds to the available potential energy of the stream and is not a consumer of potential energy.

Maddock (1970) used the criterion of the mean velocity multiplied by the slope which gives the 'falling velocity' of a body of water as a criterion and stated :

> "If v.s (mean channel velocity times slope) does exceed ω, (the fall velocity of the particle) the particle adds to the energy of the stream rather than being a burden on it, which makes the determination of the effect of small sizes of sediment particles extremely difficult."

8.4 A conceptual puzzle

Initially I had a conceptual problem with the use of v.s as a criterion for determining whether or not sediment adds to the potential energy of the water body. Consider a fine grain of sand in a jar of water. It will settle to the bottom of the jar. Now consider whether the grain of sand will reach the bottom of the jar if the jar is taken down the office lift which travels faster than the settling velocity of the particle.

If you have resolved that problem consider what would happen if you used a steel ball instead of a grain of sand and you the dropped the container down a mine shaft.

Now consider what happens to a table tennis ball which starts off at the bottom of the jar which is dropped down a mine shaft.

Lastly, which jar would wreak the greater havoc at the bottom of the shaft - the one with the steel ball or the one with the table tennis ball ?

PS What about a steel ball in a jar carried in a Mirage jet diving towards the ground at full throttle ?

If you feel that this puzzle is trivial bear with me for a while because in the sections which follow we will be considering aspects of river mechanics where the state of the art is such that even the visualisation of the processes is still in dispute.

8.5 The mode of sediment transport

The role of sediment transport in river flow will be discussed in more detail later. In the meantime all that is needed is an appreciation that at the one end of the scale very fine colloidal material adds to the driving forces which mould the shape of rivers, while at the other end of the scale large boulders in river beds result in a large increase in rotational energy conversion without being moved by the flowing water. In between these limits there is a continuous gradation of sediment particle size. Depending on the flow conditions (primarily flow velocity), this sediment will remain in suspension over long distances, or roll continuously along the bed, or move in leapfrog fashion from dune to dune, or perform a succession of these feats.

9. DEGREES OF FREEDOM

9.1 The simplest case - one degree of freedom

So far we have been considering the simplest possible case - that of steady flow in a long, uniform, straight channel of fixed slope and bed roughness. Under these conditions the river could only respond by adjusting its depth of flow (and thereby its mean velocity and cross-sectional area). If the flow, cross-sectional shape, slope, and roughness of the channel are known, the resultant depth of flow (and consequently velocity) can be calculated uniquely by using well-known energy equations (eg. Manning).

This can be shown schematically as follows :-

Driving variable	:	Flow rate (steady)
Control variables	:	Channel shape (straight, uniform) " slope " roughness
Response variables	:	Depth of flow } uniquely related Velocity of flow }
Degrees of freedom	:	One
Equations required	:	One (eg. Manning)

9.2 Introducing a mobile bed

Now consider the case which is exactly the same as that above except that the bed (but not banks) of the channel consists of fine sand. If the channel is steep enough it can take some of the sand which constitutes the bed into suspension or move it along the bed. For continuity, assume that the flow entering the reach has the same sediment concentration as that leaving it.

In order to assist in visualising the process, assume that the difference in elevation between the entrance and exit to the reach remains constant (ie. the potential energy per unit volume of water remains constant), but that the slope is increased by decreasing the length of the reach. As the channel length is decreased, the total area of the channel bed within the reach is also decreased, and therefore the available potential energy expressed in terms of unit channel bed area will increase as the slope increases.

For a very long channel, the slope will be small, and the velocity low. If the velocity (and therefore stream power per unit area of channel bed) cannot move the sand particles, the effective roughness will be the same as that of a fixed bed.

As the channel length is decreased, the velocity will reach a value where individual grains of sand start rolling along the bed. However, 'corrugations' will soon start forming in the bed in the form of ripples. As the velocity increases, larger dunes of sand form in the river bed. It has been observed that the dimensions of these dunes are independent of the depth of flow. The dunes are formed of bed material. Individual grains of sand move up the upstream face of the dune and are deposited on the lee slope where rotational flow causes the water to flow in an upstream direction behind the dune.

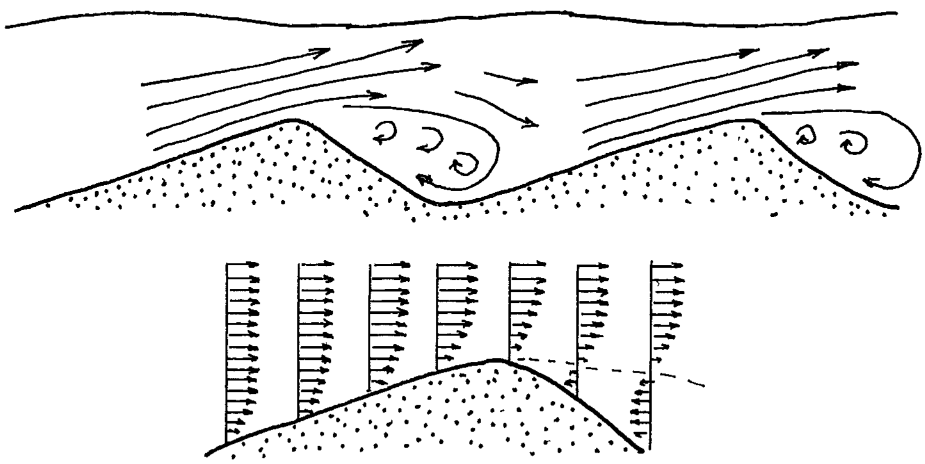


FIGURE 26 : FLOW PATTERN ASSOCIATED WITH DUNE FORMATION (AFTER ALLEN 1965)

As the velocity continues to increase a stage will be reached when a sudden change takes place. The dunes will be obliterated and the grains will move in continuous motion in a relatively deep sand/water layer along a flat river bed.

When the velocity exceeds critical velocity ie.

$$\frac{\bar{v}^2}{gD} > 1$$

then dunes start forming again, but this time the sand is removed from the downstream face of a dune and deposited on the upstream face of the succeeding dune. In this way the dunes themselves are either stationary or move in an upstream direction and are called anti-dunes. The water surface is in phase with the dunes on the bed.

(See Figure 27)

9.3 Effect of sediment transport on flow resistance

Many equations have been developed in attempt to relate flow and sediment transport, but still serious differences of opinion on the basic processes remain. Two quotes illustrate this.

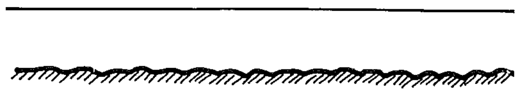
Rooseboom quotes ven te Chow as stating :-

> "The suspended material and the bed load, whether moving or not moving would consume energy and cause head loss or increase the apparent channel roughness."

Rooseboom considers this statement to be incorrect with which I agree.

However, when entering the arena on the subject of whether or not bed load should be differentiated from suspended load, Rooseboom sides with those who maintain that there should be no differentiation and that it should be possible to have a single formula which represents the total carrying capacity of a stream. He supports Yang's equation which has the form :-

RIPPLES (1)



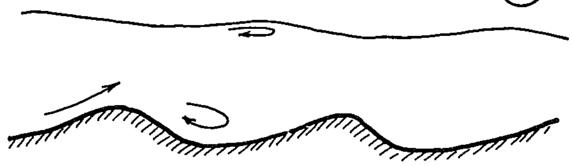
Froude number $\frac{v}{\sqrt{gd}} \ll 1$

Height: 6 - 60 mm; independent of depth

Flow resistance: high; function of depth; independent of grain size

Bed load: 0 - 0,01 %

DUNES (2)



Froude number < 1

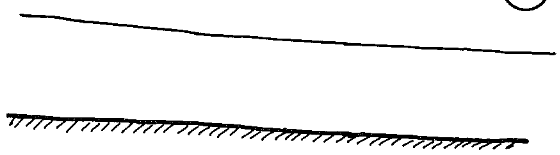
Height: up to 13m (Mississippi); amplitude = average water depth

Flow resistance: high; independent of depth; dependent on grain size; dependent on form roughness

Bed load: 0,01 - 0,12 %

Mode: erosion upstream and deposition in lee of dune

PLANE BED (3)



Froude number: 0,3 - 0,8

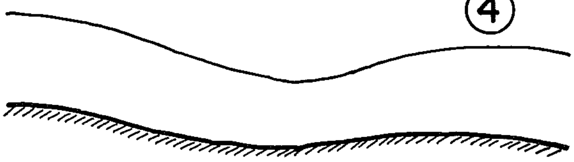
Height: largest grain size

Flow resistance: low; independent of depth; dependent on grain size

Bed load: 0,14 - 0,4 %

Mode: continuous movement

STANDING WAVES (4)



Froude number > 1

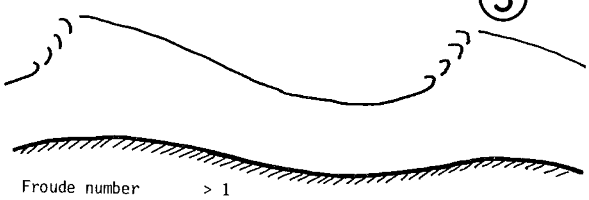
Height: up to 2m; dependent on depth; 0,5 - 0,7 x surface wave

Flow resistance: low but > plane bed

Bed load: 0,4 - 0,7 %

Mode: continuous movement

ANTI-DUNES (5)



Froude number > 1

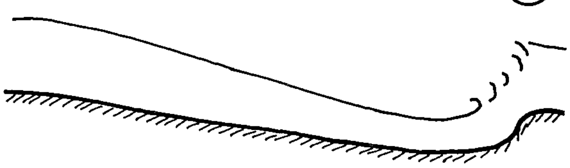
Height: up to 2m; dependent on depth; dependent on grain size

Flow resistance: high due to breaking waves

Bed load: 0,6 - 1,5 %

Mode: erosion of downstream face and deposition in front of next dune

CHUTES AND POOLS (6)



Froude number $\gg 1$

Dimensions: much longer than previous cases

Flow resistance: large

Bed load: 0,6 - 1,5 %

FIGURE 27 : HYDRAULIC AND SEDIMENT TRANSPORT PROPERTIES OF BED FORMS

$$log \frac{qs}{q} = \alpha \, log \, \bar{v} \, . \, s + \beta$$

where qs and q are the sediment and water flows, $\bar{v}$ the mean velocity, s the slope, and α and β are coefficients

The problem with this equation is that $\bar{v}$ and qs are both response variables. The mean velocity will depend on the bed roughness which depends on the rate of sediment transport which depends on the mean velocity.

Rather than taking sides at this early stage, let us try to visualise the process.

In the case of the fixed bed roughness situation, we noted that the degree of roughness determined how the total available energy was divided between maintaining translational (forward) and rotational flow respectively. This in turn determined the mean down-channel velocity.

Now a third energy-demanding variable has been introduced, namely sediment transport. To complicate matters the bed roughness also changes. As there are now two degrees of freedom, we require two simultaneous equations to solve the problem.

Neither the modified Chézy nor Manning-type equations are valid where some of the energy is being used to transport sediment.

Let us leave this question in suspension and proceed straight to the extreme condition where all the control variables are free to respond to the single driving variable - steady flow.

10. ENERGY DISTRIBUTION IN A RIVER CHANNEL

10.1 The process variables

Consider the situation where a river enters a coastal valley through a 'poort' with solid bed and flanks which fix the position and elevation of the river as it enters the valley. The only other control is mean sea level. There are no further lateral or vertical controls so the river is free to take any course down the valley. For the time being assume that there is a steady unvarying flow of water in the river and that the sediment load entering the valley is the same as that washed out to sea.

Driving variables	:	Water flow rate (constant) Sediment flow rate (constant equals sediment discharge from reach)
Control variables	:	Difference in elevation between entrance and exit.
Response variables	:	Channel shape (width/breadth ratio) Channel length (slope) Channel sinuosity Water velocity Sediment flow rate

When entering this controversial sea of opinions we can avoid many of the pitfalls which hindered the thoughts of so may others who have considered it before us by remembering that for a given flow and fixed difference in elevation between entrance to and exit from the valley the total available potential energy is constant - *the only problem is how this energy is distributed among the energy demanding processes along the course of the river.*

10.2 Energy demanding processes

Energy is consumed (converted to heat energy) by the following processes most of which are interdependent.

A. *Translational movement*

(1) Maintaining forward flow by overcoming the shear resistance within the water body.

(2) Maintaining the net forward movement of sediment by overcoming the gravitational attraction and friction resistance of the sediment particles.

B. *Rotational movement*

Maintaining rotational movement of water by overcoming shear resistance within the water body. Rotation is caused by the following factors :-

(3) Bed form roughness
(4) River bank roughness
(5) Sudden changes in the cross-sectional area of the channel (lateral as well as vertical)
(6) Changes in the direction of flow
(7) The differential mean velocity of separate flow zones such as during overbank flow.

C. *Erosion and redistribution of bed material*

A river with an appreciable sediment load flowing through alluvium is never in a static equilibrium even during the assumed condition of constant water and sediment flows. The material which constitutes the channel bed is itself in the process of being conveyed downstream. There is a continuous process of erosion and redistribution of sediment, and therefore consumption of energy, as the river maintains its optimum hydraulic properties. These include :-

(8) Creating and maintaining equilibrium bed form roughness in addition to maintaining the forward movement of sediment (see (2))

(9) Creating and maintaining the equilibrium width and depth while the meander belt migrates downstream.

(10) Creating and maintaining the equilibrium channel slope (ie. maintaining channel sinuosity) by differential erosion and redistribution of sediment along the channel banks while the meander belt migrates downstream.

10.3 Energy distribution

Given a constant water flow and sediment load plus fixed difference in elevation between the valley entrance and mean sea level, the total rate of energy expenditure with time is constant and independent of all the processes and constraints along the river course.

There are at least the ten processes listed above which require an allocation of the available energy in order to maintain the system in equilibrium. The shape of the channel, from bed roughness right through to the width of the flood plain itself will depend on how the available energy is distributed among these processes.

10.4 Time dependence of formative processes

From the preceding paragraph it is clear that energy is required not only to sustain the movement of water and sediment through the system, but also to maintain the equilibrium hydraulic properties of the river channel.

In the real life situation both the water flow and the sediment load vary over a wide range with time. The water/sediment ratio also varies. The system is therefore in a continuous state of tending towards an equilibrium condition associated with the water and sediment load at that moment, but which changes from day to day.

Some of the processes such as sediment transport and changes of bed forms from one type to another respond rapidly to changing flow conditions and can keep pace with these changes.

At the other extreme, processes such as the maintenace of the channel slope can only respond over a long period of time - possibly measured in decades - so they will reflect long term changes that take place over many seasons.

The comprehensive investigations by Ackers and Charlton (1970) at the Hydraulics Research Station in the United Kingdom illustrate this time dependence very well.

They cut a straight trapezoidal channel down the centre of a sand filled strip about 100m long and 10m wide :

> "Immediately after the introduction of water and sediment at a constant rate into the top end of the channel the bed developed ripples which were symmetrical about a longitudinal line. After a period of 6 to 24 hours, the symmetry deteriorated and shoals began to form near the sides of the channel along its length. These were at fairly regular intervals, the shoals forming on alternate sides of the channel with deeps near the opposite bank. The shoals continued to increase in length and height while they migrated fairly rapidly downstream increasing their pitch as the channel become wider through general bank erosion even though it remained straight. Then, almost simultaneously along the channel, embayments were eroded in the banks opposite each shoal. These expanded, producing a sinuous channel pattern in plan, and also migrated downstream, although more slowly than the shoals had done."

10.5 Observations of the Orange River at Oviston

The rapidity and severity of the response of bed and channel forms to changing flow was brought home to me during the severe flood in the Orange River of February 1967. An extensive but relatively flat point bar had been developed across the channel by previous flows in the river.

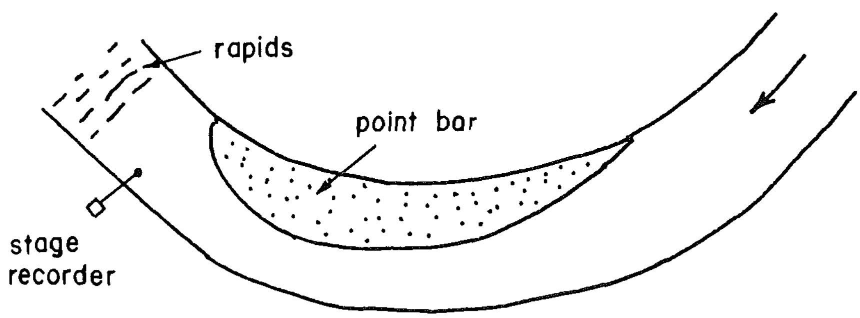


FIGURE 28 : LOCATION OF THE POINT BAR IN THE ORANGE RIVER AT OVISTON.

During the rising stage of the flood an impressive train of breaking anti-dunes developed over the point bar area. The sediment in the point bar was clearly being removed at a very high rate. Within hours the anti-dunes subsided indicating that the bulk of the sediment had been removed and that the water velocity had dropped to sub-critical velocity as the depth of flow increased.

There was a distinct hump in the water surface at the centre of the river channel at this time which was also reflected in the difference in elevation between the water level in the well of the stage recorder and that of the gauge plates on the bank of the river - something I was to observe again and photograph at a bridge across the Orange River near Keimos several years later.

11. PRIORITIES IN ENERGY DEMAND

11.1 Water discharge

No matter how small the difference in elevation, water will always flow towards the lowest point. In the extreme case the flow velocity will be so slow that the river will be unable to move a grain of sediment, and certainly not able to adjust the shape of its cross-section or course. In this situation almost all of the available energy will be used in maintaining forward movement of the water by overcoming the viscous shear in the immediate vicinity of the channel perimeter. Velocity will be constant throughout the rest of the water body, and as there will be no velocity gradient there will be no shear stress and therefore no other energy consumption.

11.2 Sediment discharge

Similarly, when a system is in equilibrium the sediment load leaving the system must be equal to that entering it. This must be so because if the hydraulic properties of the channel be such that it cannot transport the sediment load entering it, then the sediment will be deposited in the upper reaches of the channel and will continue to do so until the channel slope is such that a velocity is reached which is sufficient to transport the incoming sediment through the system.

The transport of sediment therefore has second claim on the available potential energy.

11.3 Creation of bed forms

In the low velocity range, the individual sediment particles are propelled along the river bed, but ripples are soon formed such that sediment is scoured from and carried up the upstream slope of a ripple and deposited on the lee slope. As the velocity increases large dunes are formed in the river bed by the same process.

The transport of bed load sediment therefore cannot take place without the deformation of the bed into a ripple or dune field which imposes another energy demand.

A river flowing in alluvium must at all times and at all points along its course transport the material in the river bed at a rate which maintains the system in equilibrium. If there is an increase in velocity, the following reactions will follow :-

(i) The shear force along the bed will increase and more material will be taken into suspension. This will reduce the energy available for the forward movement of water and so will tend to reduce the velocity. This will stabilise the velocity/transport rate on its own even if no other processes are involved which is not the case.

(ii) A secondary result of the increase in sediment movement is that the bed resistance to flow changes. It may increase or decrease the translational velocity depending on the initial state.

(iii) Another secondary effect of the increase in sediment movement is a change in either the magnitude or the type of bed form. This can also either increase or decrease the rotational velocity.

Reactions (ii) and (iii) are stable if they tend to reduce the proportion of potential energy available for maintaining translational velocity, but the process becomes unstable when the reverse occurs. This happens when critical velocity is approached. Dunes are washed out by the higher velocity and sediment is transported in a continuous sheet close to the bed (see para 9.2)

When sediment is being transported, the fast response processes achieve the equilibrium condition by maintaining a balance between the relative proportions of the total potential energy used for :

(i) Maintaining water transport (translation and rotation)

(ii) Maintaining sediment transport.

(iii) Maintaining rapid response hydraulic characteristics.

11.4 Slow response channel forms

Helicoidal flow is responsible for the selective erosion and deposition of sand bars on alternate banks of the river, while the reversal of the direction of rotation of the main helicoidal component causes (or is it the consequence of?) the development of meanders which are the energy demanding characteristics which have the longest response time.

11.5 Relationships between discharge, bed forms, and relative energy consumption

The three diagrams in Figure 28 are my schematic visualisation of how the available potential energy is consumed during the various processes which take place within a river channel. I have shown the relationships as being linear in the absence of information on their actual form. The relative proportions are diagramatic only for the same reason.

The point 'a' marks the discharge at which transport of bed material starts. This must be concurrent with the formation of ripples and then larger dunes as discharge increases. Between 'a' and 'b' bed form size increases and a greater proportion of the available potential energy is converted into rotational energy. This is evident by the increasing magnitude of the 'boils' of water that dominate the water surface pattern at high flows.

When the discharge reaches point 'b', the bed forms can no longer withstand the increased tractive force associated with the increased velocity. The bed forms collapse causing a sudden decrease in effective channel roughness, and consequent decrease in rotational energy demand, and increase in translational energy. As a result there is a sudden further increase in velocity and decrease in depth of flow.

At point 'c' the discharge is such that the velocity equals the critical velocity given by :-

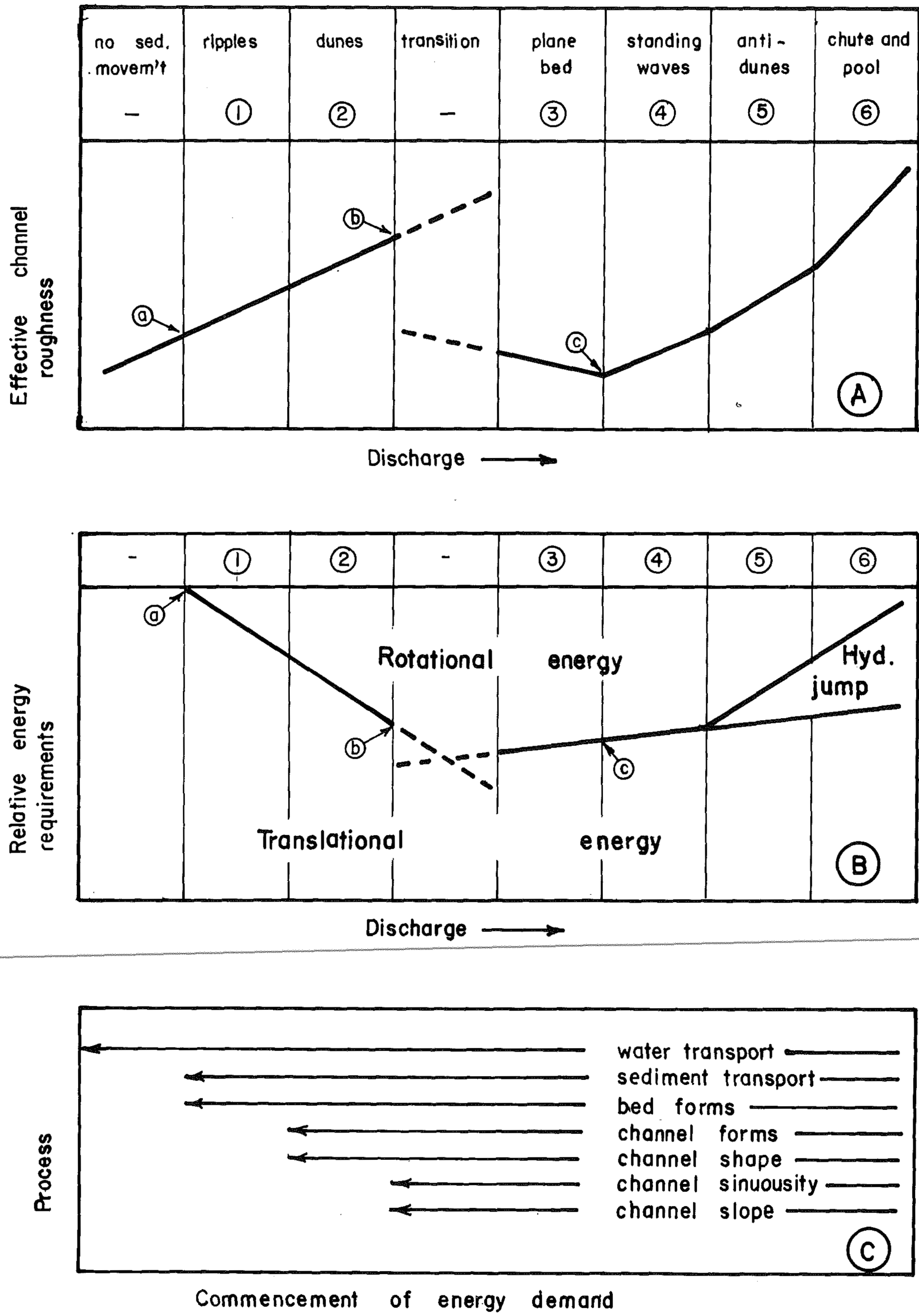


FIGURE 28 : DIAGRAMATIC REPRESENTATION OF RELATIVE ENERGY DEMANDS.

$$\textit{Froude number} = \frac{\bar{v}}{\sqrt{gD}} = 1$$

Above this discharge anti-dunes are formed. When these break, additional energy increases linearly with increasing discharge. Figure 28 C shows how additional energy demanding processes come into play as the discharge increases. The representation is diagramatic only.

11.6 A puzzle concerning the rate of sediment transport

This is an opportune moment to return to the subject of sediment transport which we left in obeyance in paragraph 8.5.

One aspect of sediment transport that has been badly neglected in the literature is the question 'how does a river maintain an equilibrium under conditions of varying flow rate and varying sediment concentration ?'

Few will contest the following statements :-

(i) A river flowing in alluvium will at all times transport the material which constitutes the river bed at the maximum potential rate associated with the prevailing hydraulic conditions.

(ii) This maximum rate is strongly related to the rate of water flow, and increases exponentially with river flow.

(iii) The suspended sediment load of a river (and therefore sediment concentration) varies over a wide range during a single flood as well as from year to year.

It is the last statement which introduces the puzzle. Seemingly there must be active aggradation and degradation of the channel bed during the passage of a single flood and from season to season.

A little more thought must bring one to the conclusion that there can be no long term equilibrium if the transport of sediment is a function of water flow alone. As many rivers are in equilibrium despite wide changes in sediment concentration the three statements above must be incomplete.

In my view the answer to the puzzle is simple. The reason why it has not been given the attention it deserves is the wide, and to a large extent unavoidable, discrepancy between the artificial conditions in a laboratory and the actual conditions in the field.

Most laboratory studies have been carried out in straight flumes with uniformly graded sediment.

Just as no river follows a straight path so too is the material in a river bed never uniformly graded. Although the average size of the bed material will decrease down the length of the river, branch streams will generally introduce material which is coarser than that in the main channel. Bank erosion is a further source of coarser material. To some extent the river will adjust its gradient at stream confluences where the two streams have significantly different sediment concentrations and grain size distributions.

A single flood does not sweep the river channel clean of even the finer sediment. The reason for this is that bed load material is not in continuous state of movement. Even during high floods this material will spend a large proportion of the time buried in bed forms and channel bars, and therefore its net rate of downstream movement is considerably less than that of water flow.

At any one time the river bed will consist of a fairly wide range of sediment sizes. These will not be uniformly mixed but will be segregated by the non-uniformity of the velocity at the cross-section as well as the different modes of sediment transport that often occur within a short reach of the river.

11.7 Sediment sorting in alluvial channels

Rana, Simons and Mahmood partially rectified the neglected studies on the effect of sediment size in 1973. They stated :-

> "The hydraulic sorting of bed sediments primarily occurs through differential transport of sediment sizes forming the bed material. As a result the sorting phenomenon is affected by all the variables involved in the transport phenomenon. These variables include the channel discharge, energy gradient of flow, bed forms, regime of flow, bed material discharge, and properties of the channel, fluid, and the bed material ... local sorting occurs over longitudinal distances smaller than a bed form length, and the progressive sorting takes place over distances many times larger than the length of the bed forms. Local sorting can be further divided into two parts related to the mode of transport, ie. due to the transport as suspended load and as bed load. .... in natural channels, a number of extraneous variations in bed material size are superimposed on the size changes caused by progressive sorting. These extraneous size variations are caused by the introduction of sediments with different sizes and gradations from tributary inflows and bank erosion ...."

> "Another difficulty in the study of longitudinal sorting is related to the separation of cause and effect in alluvial channel phenomena. Both the energy slope and bed material size generally decrease in the downstream direction of an alluvial channel. In such instances, it is difficult to decide whether the slope is the cause or the effect ....."

While these authors were very much aware of the influence of non-homogeneity of bed material on channel characteristics they did not cover this aspect in their paper.

Also in 1973 Ackers and White proposed a new approach and analysis on sediment transport. Their paper is very interesting but they acknowledged that "... Further work is, however, required to study the effects of graded sediments and of unsteady flow conditions."

11.8 Sediment transport equations

Wash load is dependent on the rate of sediment production within the catchment and is independent of river flow. It can therefore not be included in sediment transport equations. Sediment transport equations for the bed load can at best only be expected to give approximate results because of the following :-

(i) The variability of the sediment sizes at a given section of the river.

(ii) The complex relationship between bed form and sediment transport rate, including the discontinuity when critical velocity is approached.

(iii) The armouring effect of the larger sizes which protect the finer material from erosion.

12. PROPERTIES OF A MEANDERING RIVER

12.1 Meander dimensions

By now it will be apparent that rotational flow is present in all but the slowest moving water. Secondly no river outside a laboratory ever starts its life flowing in a straight course. We should therefore not be surprised to find that rivers follow an irregular course, but the question remains why does a river flowing in alluvium so often follow a regular sinousoidal meandering course ?

Leopold and Langbein (1966) made the succinct observation that :-

> "Meanders will usually appear wherever the river traverses a gentle slope in a medium consisting of fine-grained material that is easily eroded and transported *but has sufficient cohesiveness to provide firm banks.*"

The italics are mine. This is a very important qualification as will be seen later.

First of all, some definitions :-

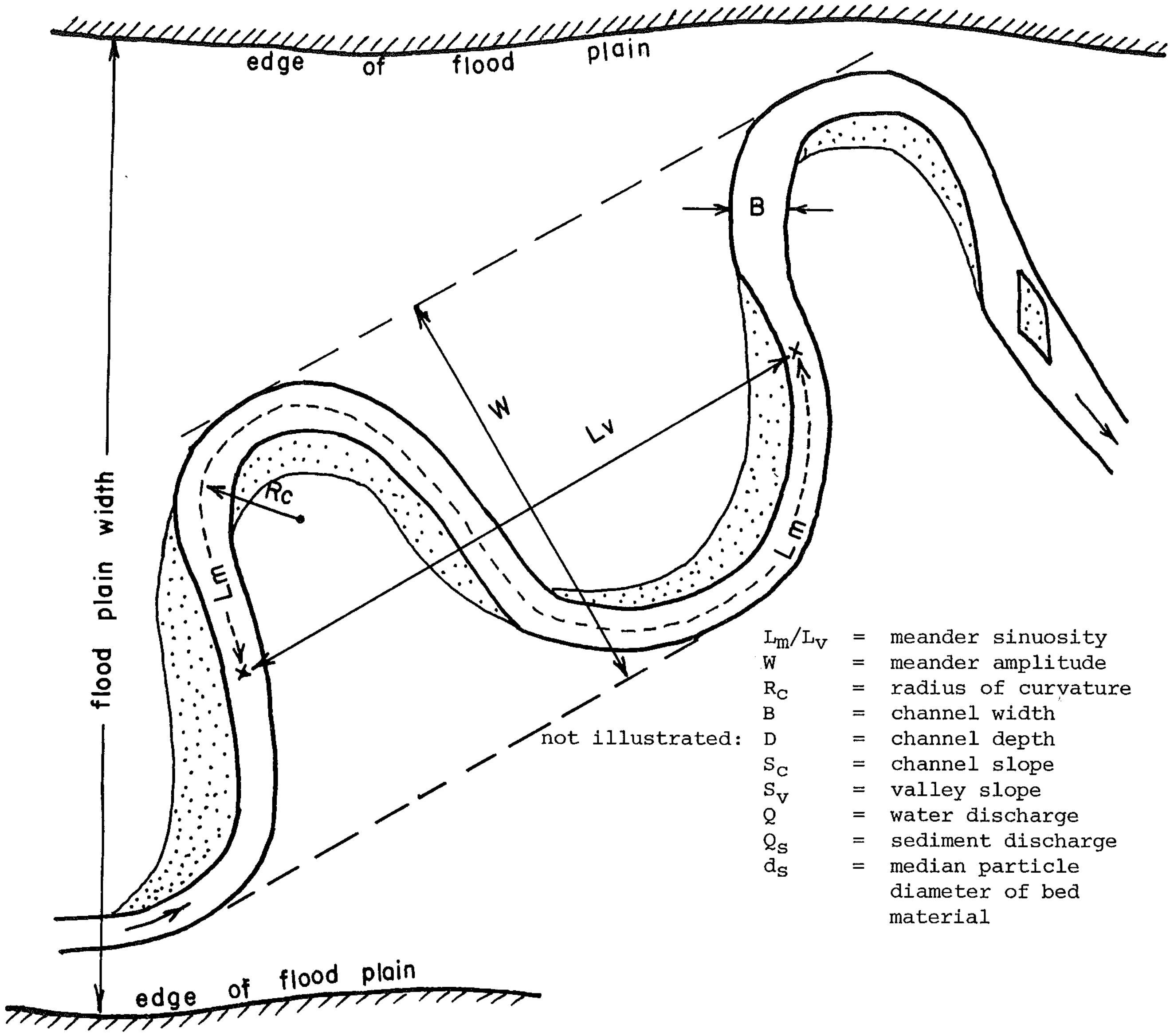


FIGURE 30 : MEANDER PATTERN GEOMETRY.

The main dimension of interest is the extent to which the river course deviates from a straight line. The accepted term for this is channel sinuosity although this does not necessarily imply that the shape is that of a sine curve. Some authors use the term tortuosity but this implies irregularity which is also not what we have in mind.

Sinuosity is determined by dividing the thalweg length (thread of deepest flow) by the straight distance between two points which mark the limits of a complete meander cycle.

Channel slope and valley slope are easier to conceive than to measure. The other dimensions are straight forward.

12.2 Regime theory

In the context of regime theory, earth canals with known cross-sectional shapes, water and sediment flows, and bed material have three degrees of freedom - flow depth, water surface width, and adjusted slope. A fourth degree of freedom exists in river channel flow - the freedom to meander. A river or canal which had attained an equilibrium condition was said to be 'in regime'. Empirical relationships were developed from observations of canals and rivers that had reached an equilibrium condition and these relationships were applied when irrigation canals were constructed.

These properties of canals and river channels were studied extensively just before the turn of the last century when irrigation engineers in India were faced with the problems of scour and sediment deposition within the large unlined irrigation canals. The engineers were unable to develop theoretical relationships between the principal dimensions and characteristics shown in Figure 30, and had to fall back on an analysis of observed relationships. For example there is a very good correlation between channel width and meander sinuosity, but poor correlation between sinuosity and the other variables.

Professional Paper No 5 issued by the then Department of Irrigation (now Water Affairs) in 1919 was based on a paper by Kennedy in 1895. The title of the Professional Paper was "Slopes for flood furrows from very muddy rivers." The introduction to the tables which followed was a very pragmatic approach to the subject :-

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"In choosing between a steep and a gentle slope the following considerations must be weighed against each other.

For the steeper slope there is a gain in smaller sectional elevation, also a saving in annual silt clearance and probably a gain in fertilising value on the land. On the other hand, headworks may have to be higher and more expensive, or a smaller area of land commanded and there may be additional cost in repairs due to scour.

When the good land is at a level and in excess of requirements, and when the furrow can be passed over rock at intervals and silt is beneficial for the land, then the steeper slope which will carry silt and sand along has every advantage.

We have very little information as to what is the gentlest slope that will carry sand and silt. The most extensive results are those of Kennedy in India. He found that the critical velocity (Vo) which would neither silt nor scour was a function of the depth only, and could be expressed for certain river by $0.84d^{0.64}$ (d = depth). It should be observed that (1) the smallest depth observed was 2.2'. The low values of Vo below this depth do not seem to be reasonable; (2) that he was dealing with canals which run throughout the year, and a certain amount of the self cleaning was done in low water periods when the silt charge was low. In flood rivers this cleaning period is much shorter. (3) The silt and sand were probably not as coarse as in some of the South African rivers.

For these reasons the formula has been modified so as to give higher values of Vo, and in such a way as to produce a relatively greater increase for small depths."

"In the attached tables the following other limiting conditions have been adopted :-

D	discharge		between 3 and 500 cusecs
S	slope	S	" 1/500 and 1/3000
d	depth	d	" 1 ft. and 5 ft.
b	bottom width	$\frac{d}{b}$	not greater than ½ and not less than 1/10.

Side slopes of 2 on 1 have been adopted and Kutter's 'N' = 0.025.

For each depth the minimum velocity as limited by these considerations is indicated above by a dotted line and the maximum by the full lower line."

The empirical relationships derived from regime theory have to be used with care when applied to conditions significantly different from those studied. Nevertheless they do give a useful first estimate of the likely values.

12.3 Law of uniform energy loss per unit channel length

In 1966 Leopold and Langbein, Chief Hydrologist and Research Hydrologist respectively of the U.S. Geological Survey stated :-

> "It was as if the river had, to use somewhat anthropomorphic terms, chosen to cut a meander curve in order to achieve a more uniform water surface profile. This suggested that the river had chosen the curved path in order to achieve the objective of uniform energy loss for each unit distance along the channel, *but had paid a price in terms of the larger total energy loss inherent in a curved path.*"

Once again the italics are mine. As we have seen the loss in potential energy measured down the valley is independent of the length of the channel. The statement in italics is therefore incorrect - there is no change in the total energy loss, and thus no price to be paid. What the increased curvature does is to decrease the rate of energy expenditure per unit length of channel. The greater the channel length the less the amount of energy available per unit length of channel and therefore less energy per square metre of channel perimeter, therefore less energy for erosion and redistribution of sediment.

These two authors also likened channel sinuosity to the shape that a strip of spring steel takes when it is bent. This shape is such that the energy applied in deforming the steel is uniformly distributed along its length. They then demonstrated that sine-generated curves using a mathematical random walk technique had the property of minimising the sum of the squares of the changes in direction and that this is also the curve of minimum total work in bending.

12.4 An impossible task

In 1970 Maddock a research hydrologist in the United States Geological Survey stated :-

> "Much effort has been expended in attempting to establish determinate relations for the solution of the dual problems of resistance to flow and sediment transport in alluvial channels. The thesis herein is that this is an impossible task."

Despite his pessimism he did draw some pertinent conclusions; which I have paraphrased as follows :

(i) Field and laboratory data indicate that the relations among width, depth, velocity, and slope are indeterminate unless the constraints on the development of bed forms are known.

(ii) Although the hypothesis that each bed form has a characteristic rate of transport is true, this evaluation is deficient when it does not recognise that bed form is also the mechanism by which dynamic equilibrium is maintained.

(iii) Bed form is the most quickly and easily adjusted dependent variable in natural and artificial channels. In both types of channel, but particularly in rivers, local and sometimes short-lived changes in bed configuration are the means by which dynamic equilibrium is maintained, (see also para 10.5 above).

(iv) In alluvial channels, width is more readily adjustable than slope, but the adjustment is slow, particularly when a trend toward reduction in width is required. Furthermore, *width is subject to almost catastrophic change in times of floods of unusual magnitude and duration* (my italics and my experience as well !)

(v) The combination of a quickly adjustable bed form, slowly or erratically adjustable width, and uncertain adjustable slope is accompanied by unpredictable behaviour of natural stream channels. Thus, the behaviour of natural alluvial channels cannot be predicted from flume studies or by any other approach, and, it is impossible to forecast precisely what will happen at any one point, in place or in time, in any channel with a movable bed.

(vi) Because it is impossible to have an alluvial channel that will remain in equilibrium under a wide range of discharges of water and sediment. problems associated with alluvial channels can and must be resolved only by recognising and accepting the lesser of a good many evils. (By evils I assume Maddock meant undesirable consequences of man-made intervention).

12.5 The British view

Also in 1970, Ackers and Charlton reported on their large field sand-bed experiments at the Hydraulics Research Station in Wallingford, England. And here is a summary of the points they made :

(i) Inherent in the majority of, if not all, streams are forces which oppose the maintenance of a straight channel.

(ii) In spite of various hypotheses, the true cause of meandering is still obscure. There exists no generally accepted means of predicting, under given conditions of fluid and sediment flux, whether meanders will occur, although there is considerable empirical information which is of value in predicting aspects of the geometrical pattern which may result.

(iii) Intuitively the geometrical dimensions of a meandered channel may be dependent on :-
   (a) the discharge and its variation with time
   (b) the slope of the valley
   (c) the rate of sediment transport
   (d) the resistance of the bed and banks to movement which in turn is dependent on grain size, specific gravity and cohesive strength
   (e) the viscosity and density of the liquid.

(iv) Many reports on meandering channels have been written, some being based on small scale laboratory experiments and some on the analysis of field scale data, and almost as many hypotheses have been put forward to explain why rivers meander. The purpose of the present series of experiments was to add to the fund of data in the hope that by conducting research under controlled conditions available in the laboratory greater confidence could be placed in the relevance of the results for practical conditions in nature.

12.6 Theoretical support from the USA - the law of least rate of energy expenditure

In 1971 Yang of the Illinois State Water Survey provided strong theoretical support to the experimental observations of Ackers and Charlton.

Here are some quotes.

(i) The only useful energy nature provides to a unit mass of raindrops falling on the slope of a watershed ('catchment' in South Africa !) is its potential energy above a datum say, the sea level. This potential energy is the source of energy that gives overland streamflow the ability to carve channels and form a stream network. The distribution and expenditure of this potential energy along the course of flow will determine the shape of the stream channels and the form of the stream network. Hence the shape of all streams observed today should be the cumulated result of the distribution and expenditure of potential energy along their course of flow in the past million years. From this starting point, the study of stream morphology can be simplified.

Yang then stated his law of least rate of energy expenditure :

"During the evolution towards its equilibrium condition, a natural stream chooses its course of flow in such a manner that the rate of potential energy expenditure per unit mass of water along this course is a minimum. This minimum value depends on the external constraints applied to the stream."

Note that it is the rate of energy expenditure per unit mass of water. The total energy expenditure cannot be changed.

However as we are concerned with the river channel and not the water, the law could have been reframed by subsituting "... such that the rate of energy expenditure per unit length of channel is a minimum ..."

What the law does illustrate is that as the maximum rate of energy expenditure is achieved when the river flows in a straight path down the valley, this is the least likely route that it will follow.

The difficulty in applying Yang's law is that the rate of energy expenditure decreases linearly with increase in channel length until it approaches zero for an infinitely long channel. The minimum value in practice will therefore depend on the external constraints, and we are no nearer to solving the problem until we can define and quantify these constraints.

Yang described it this way :-

"Along the path of flow, a stream will do all it can to minimise its rate of potential energy expenditure per unit mass of water. However, there are some constraints that are imposed on the stream by its environment. These constraints make it impossible for a stream to reduce its rate of potential energy expenditure per unit mass of water indefinitely. As soon as a stream's channel characteristics such as slope or width, are insufficient to carry its water discharge and sediment load, changes will take place to maintain a condition of dynamic equilibrium. Channel braiding is one way in which a natural stream can adjust itself to maintain dynamic equilibrium along its course of flow."

In a second paper also in 1971 Yang applied his law to demonstrate the influence of the change in water discharge, sediment concentration, channel geometry, channel slope, valley slope and geological constraints on the meandering channel characteristics. The study was restricted to the case of unbraided channels in dynamic equilibrium or during their evolution toward their equilibrium conditions. He started off by postulating that :-

> "... all the previous theories, hypotheses, or analyses brought to the attention of this author fail to provide a satisfactory explanation as to why a river should meander. Meandering channels always exist in spite of the wide range of variations of water discharge, sediment concentration, geological constraints, location of channel, size of channel, strength of turbulence, etc. These phenomena suggest that there must be a basic law which governs the formation of meandering channels. This law must have the ability to explain why all stable unbraided channels always follow a smooth sinuous course. It should also explain why the longitudinal streambed profile should be concave. It should apply to channels on alluvial beds as well as on ice. Furthermore, this law should explain how changes in the factors such as water discharge, sediment concentration, channel geometry, channel slope, valley slope, geological constraint, etc., should change the meandering channel characteristics."

Yang then assumed channel shape to be a segment of a semicircle and developed mathematical relationships from which he could determine how channel slope and channel geometry should change in response to changes in the independent variables which were water discharge, valley slope, and sediment concentration as follows :-

12.7 Channel slope

Yang stated that "A natural stream always adjusts it slope to provide enough kinetic energy to transport the water and sediment load."

I would rather have reworded this statement to read :-

> "In order to achieve dynamic equilibrium a natural stream will adjust its slope and thereby the rate of energy expenditure to that required to transport the water and sediment load."

This restatement is more in keeping with the way in which an additional allocation of energy is made available to overcome gravity and frictional resistance to flow of the sediment.

12.8 Channel width

Channel width was used by Yang as a parameter to describe channel cross-sectional geometry. He could do this because the cross-sectional shape was fixed (a segment of a semi-circle), and therefore channel depth and cross-sectional area were uniquely related to channel width.

For a stream having this fixed shape Yang found that a channel would have to increase its width in order to minimise the rate of expenditure per unit mass of water (or per unit channel length).

Subsequent authors were quick to point out that this was contrary to observations of real rivers. Most rivers decrease their width with increase in sinuosity. The Pongolo River is a good example. The river has a much narrower (but deeper) channel where it passes through the Makatini Flats than along its course upstream of the Lebombo mountains.

Both Yang and his critics seem to have overlooked the fact that the cross-sectional shape of a river in alluvium has two degrees of freedom and not one. From the law of conservation of mass, the product of the mean velocity, mean width and mean depth must be constant. Applying Yang's law will result in a decrease in velocity. Therefore the cross-sectional area must increase, but this does not preclude a decrease in channel width together with a much larger increase in channel depth.

Yang himself states :-

> "It has been shown *separately*" (my italics) "how important factors of channel slope, channel width, and water discharge, which affect the meandering channel characteristics are related to the rate of potential energy expenditure per unit mass of water. *All these factors are related. A change in the value of one factor will change the other factors*" (again my italics).

Its surprising that two paragraphs later Yang nevertheless draws the incorrect conclusion :-

"Equation (22) was derived for a channel with a cross-section of a segment of a semi-circle. When Equation (22) is applied to a natural stream, the coefficients and exponents may not remain unchanged. *However, the conclusion that $\Delta H/\Delta t$ decreases with increasing channel slope, decreasing water discharge, and increasing channel width should hold for natural streams of any cross-sectional shape*." (my italics)

Equation (22) was

$$\frac{\Delta H}{\Delta t} = 0{,}963\ k.n^{-0{,}6}.r^{-0{,}4}\ .s^{1{,}3}.Q^{0{,}4}\ \left[\sin^{-1}\left(\frac{W}{2r}\right)\right]^{-0{,}4}$$

where :-

$\Delta H/\Delta t$ = time rate of potential energy expenditure per unit mass of water for a reach with relief y

k = a conversion factor between energy and fall

n = Manning's roughness coefficient

r = radius of a segment of semicircular channel

s = channel slope

Q = water discharge

W = channel width

While the italicised portion of Yang's comment cannot be faulted, it does not necessarily follow that all the dependent variables will increase with $\Delta H/\Delta t$, any more than one can assume that if box 'A' has a larger volume than box 'B', then each of its dimensions will also be longer than the corresponding dimensions of box 'B'.

While I am in full agreement with Yang's law of least rate of energy expenditure I do not agree with all the conclusions he draws from it.

12.9 India's turn

In 1973 Chitale of the Central Water and Power Research Station, Poona (India) entered the debate in his paper on "Theories and relationships of river channel patterns."

Chitale used circular arcs to describe meander patterns as these were more amenable to mathematical treatment. Figure 31 shows how two parameters can be used to define sinuosity uniquely. These are the radius of curvature Rc and the displacement of the centres of curvature.

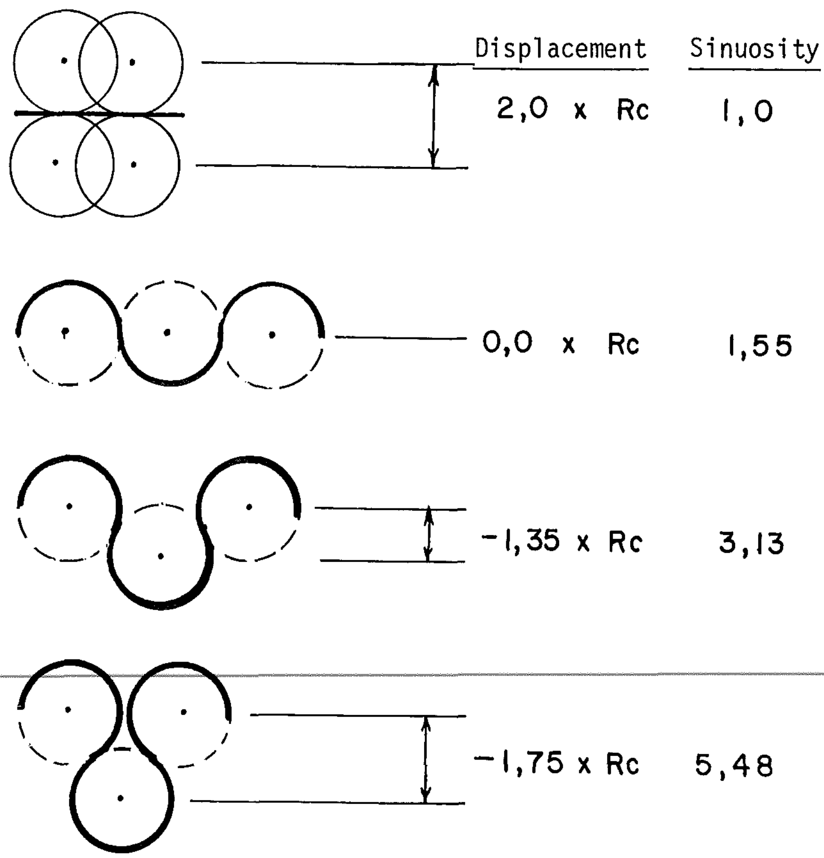

FIGURE 31 : MEANDER SHAPES DEVELOPED USING CIRCULAR ARCS (AFTER CHITALE 1973)

Unfortunately mathematical simplicity was not adequate. The theoretical maximum sinuosity of a series of circular arcs is 5,5. The sinuosity of the Collywobbles on the Mbashe River is 6,5 ! (see Figure 7)

Chitale took issue with Yang as well as with Ackers and Charlton regarding braided channels (ie. steep multiple course channels within a wide and relatively straight river channel - see Figure 32).

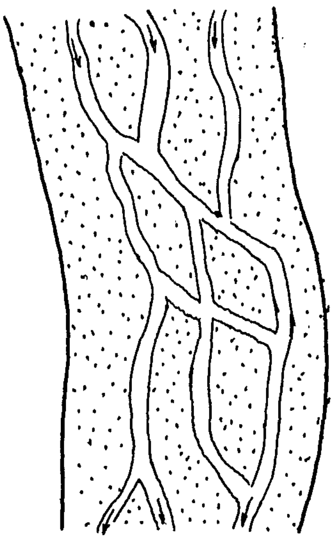

FIGURE 32 : BRAIDED CHANNEL.

Chitale correctly pointed out that braided channels are wider and steeper than meandering channels, and not the reverse as the other authors claimed. He also maintained that braided channels carry a higher load of bed material than single thread sinuous channels.

Chitale maintained that braided channels were also a consequence of readily erodable banks.

Chitale rejected Yang's law on the basis of his observations that meandering channels were narrower than braided channels, but fell into the same trap as Yang when showing this mathematically :-

In effect he argued :-

Potential energy		=	stream fall y
Rate of PE expenditure		=	y/T
Since	y/L	=	S
and	T	=	L/V
then	y/T	=	S.V

If S.V is to be a minimum, the slope and velocity should both become minimum.

Since	Q	=	V.A
	V	=	Q/A
		=	Q/b.d
and	y/T	=	S.Q/b.d

Therefore for a given discharge the requirement for minimisation of the time rate of expenditure of energy for unit mass is that slope or depth should become minimum and width should become maximum. *"Thus would imply that flatter slopes should be associated with wide and shallow streams and vice versa. Field experience, however, shows that wide and shallow streams are steeper and not flatter."*

As I explained in the previous section if the independent variable increases and there are two or more dependent variables it does not follow that all the dependent variables must move in the same direction or to the same extent.

In a process which involves a number of variables it can be most misleading to examine the relationship between the independent variable and each dependent variable one at a time while holding all the other dependent variables unchanged, and then assume that when all the dependent variables are freed they will react in the same direction.

12.10 Factors governing the width-to-depth ratio

Chitale was on much stronger ground when he stated that :-

> "The primary factors governing the width-to-depth ratio of the channel cross-section are considered to be water discharge, sediment characteristics (concentration, size and mode of transport) *and resistivity of the bank material against erosion*" (my italics). "If the stream is called upon to carry heavier concentration of bed load, it adjusts to meet this requirement by increasing the boundary shear. Higher shear stress then simultaneously causes rapid bank erosion and the channel becomes wider. In a wide and therefore shallow section a high value of sheer stress could be generated only by building up of steeper slope. Thus the wide and shallow sections are associated with heavy bed load transport, steep slopes and high velocities."

- and later :

> "Thus wide and shallow rivers are found to be associated with coarse bed material and/or heavier concentration of bed load, *less clay fraction in the bank*, etc." (my italics)

Charlton and Ackers responded by not agreeing with Chitale's interpretation of what they had said in the first instance.

12.11 Depth is a redundant parameter

In 1973 Maddock made an eye-catching statement that under certain conditions "Velocity is independent of depth in a stream supporting sediment (and) depth is a redundant parameter."

The burden of his presentation was :-

> "At some discharge in an alluvial channel the material on the bed will begin to move. As the discharge increases, the velocity may increase and with it an increase in unit sediment discharge will occur. Sediment increases with discharge until a relation exists

$$V.S.10^3 = C^{\frac{3}{4}} . \Phi(d)$$

in which $\Phi(d)$ = a function of the size of the sediment being transported and has the dimensions and characteristics of a fall velocity of a sediment particle, and C = the sediment concentration in parts per million."

He continued :

"The effect of change of slope is most interesting because the equation states that if slope is increased, velocity must decrease unless sediment concentration or size increases. ... Actually, erosion and meandering are the result of bed forms which resist flow and bring about a lower velocity. Where adequate bed forms cannot be developed, the channel simply degrades, increases its sediment concentration and decreases its slope."

I fear that Maddock has seriously confused cause and effect in this explanation, and it is little wonder that he ends off his paper with these words :-

"It is regrettable that, in ariving at the answers to our questions, so much of the existing literature (including some by this writer (ie. Maddock) will have to be consigned to oblivion."

Maddock did make another statement which is worth quoting :-

"The hydraulics of an alluvial channel is inextricably mixed with the movement of sediment, either on the bed or within the prism .... . It is unfortunate that studies of channel equilibrium, which were nearly all sand-bedded streams, ignored for a long period of time the influence of sediment concentration and size on channel width, depth, velocity, and slope."

12.12 Canada's turn

In 1974 Quick of the University of British Columbia stated :-

"The principal proposal of this paper is that meandering depends on a stream-wise spiral flow which changes its direction of rotation in a cyclical manner. .... The wave length of the meander is determined by this change of rotation.... . To produce this change of rotation it has been found necessary to identify at least three interacting mechanisms :-

1. Vorticity is generated principally by bed shears and therefore starts as a cross-channel vorticity vector....

2. Vorticity is being continuously generated in a cross-stream direction and at the same time the stream-wise component is continuously decaying .... .

3. The stream channel erodes and deposits material in response to the stream-wise component of vorticity. This entrainment and deposition of boundary material produces a deflection of the channel direction."

There have been a number of authors who have subsequently added their views to the subject of why a river meanders, but I need go no further than this paper by Quick before offering my own views. Before I do so, however, I cannot resist quoting the following paragraph from Quick's paper in full. I have not altered a single word of it.

PLEASE GO BACK TO PARA 3.5 AND READ IT BEFORE READING THE PARAGRAPH BELOW. PLEASE ACCEPT MY ASSURANCE THAT I WROTE PARA 3.5 AND THE WHOLE OF THIS TECHNICAL NOTE UP TO AND INCLUDING CHAPTER 11 BEFORE I READ QUICK'S PAPER WHICH IS TITLED "MECHANISM FOR STREAMFLOW MEANDERING" AND WAS PUBLISHED IN THE JUNE 1974 JOURNAL OF THE HYDRAULICS DIVISION OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS.

"The present study recognizes two processes for the generation of secondary flows. The first process is that considered by Einstein and Li (2), and can produce spiral flows even in a straight channel. In the second process, vorticity is generated at the river bottom in a cross-flow direction and is locked to the fluid elements in which it is generated. The fluid elements are then convected upwards into the faster moving flows and are distorted by the nonuniform velocity distribution which exists across the channel. This distortion of the vorticity vector rotates some of the vorticity into a stream-wise direction. As a subsidiary argument, the cross-stream vorticity is constrained by the shears that produce it, whereas the streamwise vorticity is free to reorganize into a more coherent spiral flow spanning the total cross-section. This latter argument is closely related to

Thomson's arguments (9,10) and Prandtl (5) who points out that secondary flows can arise from curvature of the streamlines in the presence of bed shear. The great strength of this secondary flow argument, as was recognized by Thomson, is that it explains not only the localization of scour in the cross section but also the localization of deposition, which are two essential requirements in any theory of meandering."

13. THE PROPERTIES AND FLOW CHARACTERISTICS OF NATURAL RIVER CHANNELS

13.1 Introduction

The hydraulic and geometrical properties of river channels and the closely associated properties of sediment transport have been studied for nearly a century, but only during the last decade has world-wide interest in the subject lead to a clearer understanding of these complex processes.

In these notes I do not pretend to have given you a balanced view of the state of the art. Through selective quotations I have brought you to the position where I feel that I can present a coherent, and hopefully logical, explanation of the processes which control river behaviour. In my view it will never be possible to make accurate quantitative predictions of how a river will respond to man-made intervention at a specific point along its course any more than it will be possible to predict whether or not it will rain on a specific day a year from now. Nevertheless qualitative predictions based on the most probable effects can and should be made. There is no excuse for an engineer failing to consider the likely consequence of a structure in the flood plain of a river even if he cannot calculate the consequences as precisely as he can in fixed bed hydraulic situations.

13.2 Driving variables (independent variables)

The dominant driving variables in a river system are :-

(i) ~~River flow and its variability.~~

(ii) Sediment availability including the grain sizes of the sediment delivered to the river channel and composing the river bed.

(iii) The vertical difference in elevation between the surface on which the rain falls and mean sea level. The product of the river flow and this difference in elevation determines the total amount of energy delivered to and expanded within the river system.

13.3 Response variables (dependent variables)

The major response variables of interest are the quantities and rates of water and sediment flow which leave the system.

13.4 Control variables (state variables)

The following control variables may also be considered as response variables if it is their response that is of interest.

(a) *Geometrical properties :* These include :-

(i) the cross-sectional shape of the channel (area, width, depth, shape);

(ii) the geometrical shape of the river course (straight, irregular, sinusoidal);

(iii) the slope of the river channel;

(iv) the straight-line slope of the valley in which the river flows.

(b) *Bed and channel forms :* These vary in size and shape from ripples and dunes (bed forms) through to alternate bars and point bars (channel forms).

(c) *Hydraulic properties :* The main properties of the water itself are its density and its viscosity. The velocity of the water plays a dominant role in all river mechanics processes.

(d) *River bank competence :* The competence of the river bank exercises a major control in both channel geometry and the flow processes. See Technical Note No 87.

13.5 Distribution of energy

The rate at which energy is consumed (principally converted to heat energy) within a river catchment depends wholly and solely on the difference in elevation between the land surface on which the rain falls or from which groundwater emerges and sea level, and the rate of flow. This difference in elevation is called the *potential energy* of a unit mass of water.

This potential energy is distributed among the various energy-demanding processes which occur along the river channel. The relative proportions of the total available potential energy will depend on the processes themselves as well as the total energy available (potential plus kinetic) at a particular point in the river at a particular time.

The key to river mechanics problems is the understanding of the factors which control the relative proportions of the potential energy delivered to the system among the various energy demanding processes.

13.6 Energy demanding processes

Energy is required not only to sustain the movement of water and sediment through the system but also to maintain the equilibrium hydraulic and geometrical properties of the system.

The following are the main energy demanding processes of a river system in dynamic equilibrium (see para 10.2).

1. Maintaining translational flow by overcoming the viscous shear resistance of the water.
2. Maintaining the net downstream movement of sediment by overcoming the gravitational attraction and frictional resistance of the sediment particles.
3. Maintaining the rotational movement of water by overcoming viscous shear resistance of the water. The flow will rotate in addition to its net forward movement wherever one or more of the following conditions are present :-
   (a) Bed-forms.
   (b) River bank irregularities.
   (c) Sudden changes in the cross-sectional area of the channel (lateral as well as vertical).
   (d) Changes in the direction of flow.
   (e) Separate flow zones such as main stream and overbank flow.

4. Erosion, suspension and redistribution of material in the river bed while :-
   (a) Maintaining the equilibrium bed form roughness in addition to maintaining the net downstream transport of sediment.
   (b) Maintaining the equilibrium width, depth and cross-sectional shape of the channel while the meander belt slowly migrates downstream.
   (c) Maintaining the equilibrium channel slope (ie. maintaining sinuosity by differential erosion and redistribution of sediment along the channel banks) while the meander belt migrates downstream.

As the flow in the river as well as the sediment load (and sediment concentration) may vary rapidly over a wide range, the river is seldom if ever even in dynamic equilibrium on a time scale measured in days, seasons or years, although in the absence of intervention by man it may be in long-term equilibrium measured in decades.

A river is therefore in a continuous state of adjusting from the results of all the flows up to that time towards, but never reaching, the equilibrium conditions associated with the flow and sediment load at that moment.

13.7 "Instantaneous equilibrium"

The distribution of the available energy among the energy demanding processes will be modified by the current state of the control variables. The degree of modification will be governed by the extent to which the current state differs from the equilibrium condition. An example was given in para 10.5 where a large proportion of the available energy was consumed in removing a point bar which approached an equilibrium condition for low flows but which obstructed the river channel during the large flood. Once the point bar had been reduced, critical flow which is a vigorous consumer of energy subsided to sub-critical flow and the available energy was re-distributed among other processes.

At all times the total available energy will be distributed among the various energy demanding processes. Only the relative proportions will change depending on both the current and equilibrium conditions.

The total energy available and the total energy consumption will therefore always be in balance.

13.8 Priorities in energy demand

The actual 'allocation' of energy will depend on the rapidity with which the processes can respond to changes in energy input (principally change in flow). I would place the response times and therefore priority in allocation of a change in energy input in the following order :-

1(a) Maintaining translational flow.
(b) Maintaining rotational flow.
2. Transport of wash load material.
3(a) Transport of bed material.
(b) Maintaining bed forms associated with bed material transport.
(c) Maintaining hydraulic jump where critical velocity is exceeded.
4. Maintaining channel forms such as alternate bars and point bars associated with helicoidal flow.
5. Maintaining the cross-sectional shape of the channel associated with the current hydraulic conditions.
6. Maintaining the channel slope associated with the long-term flow conditions.

13.9 Energy for translational flow

A change in the flow velocity will be the first response to an increase in flow in the channel.

13.10 Energy for transport of wash load

The transport of the available wash load will take place as soon as the turbulent flow component can support it. Thereafter the rate of transport of this material will depend on its availability and will be independent of the flow conditions.

13.11 Energy for transport of bed material

The transport of bed material and the associated energy requirement for maintaining the corresponding bed forms will have third call on the available energy. When this threshold has been passed the proportion of the total energy available for translational movement per unit mass of water will decrease but the total per unit area of wetted perimeter will increase thus the translational velocity will continue increasing.

A sudden change takes place when the velocity approaches critical velocity. The bed forms can no longer withstand the tractive force and will be wiped out. The rotational flow and its associated energy demand will decrease markedly leaving a greater allocation for translational flow as well as for the transport of additional bed material both of which will show correspondingly large increases.

13.12 Energy for hydraulic jump

Another energy consuming process is introduced at this stage - that of maintaining an hydraulic jump associated with breaking anti-dunes. This is the largest energy demanding process and stabilising factor for very high energy input.

13.13 Energy for channel forms

The helicoidal flow component of river flow and consequently the energy required to maintain it is relatively small and it does not play a major role in the energy distribution.

13.14 Energy for cross-sectional channel shape, channel slope and river geometrical shape.

River bank competence plays a major role in the allocation of energy through its effect on channel geometry. The greater the channel sinuosity the greater the energy consumed by the river in changing its direction of flow and consequently the greater the tractive force on the banks at the bends. Competent banks can withstand the tractive force. If this protection is removed the banks will be vulnerable to the high tractive force associated with the river sinuosity which itself was a consequence of bank competence. Under these conditions a large proportion of the available energy will be expended on the erosion, transport and redistribution of sediment in the river banks until a new equilibrium condition is reached. This equilibrium condition will be that associated with a wider, shallower and less sinuous channel than that of a channel with competent banks.

13.15 Why does a river not follow a straight course ?

The answer to this question is that *a straight course is the most unlikely course that a river will follow.* A straight line which follows the maximum valley slope is the course which provides the maximum rate of energy expenditure and therefore applies the maximum tractive force per unit length of channel. Any weaknesses or irregularities in the channel bed or banks will allow the water to rotate. The greater the slope the greater the translational velocity, and consequently the greater the rotational velocity and therefore the greater the scour at that point. The selective scouring will introduce rotation on a larger scale and the process will continue. With time the channel gradient will become progressively flatter due to its longer path. The translational velocity will decrease and set in motion the chain of events which will eventually stabilise the channel lengthening process when the total rate of energy within the reach of channel is just enough to maintain the water and sediment flow and bed and channel forms leaving no additional energy available for further lengthening of the river course.

Yang's law of least rate of energy expenditure is therefore valid.
(para 12.6)

13.16 Why does a river in alluvium follow a regular meandering pattern ?

The main direction of motion of a river is down the valley slope. However, when a river is deflected from this path by the processes described in the previous paragraph, the inertia of the translational and rotational flow in the new direction will tend to maintain the new direction until an opportunity arises for the helicoidal rotation to die out and reverse its sense of rotation. In this way a river will tend to have regular reversals of direction of helicoidal flow about the line of maximum slope.

Leopold and Langbein's law of uniform energy loss per unit channel length is also valid (para 12.3). For the reasons given by these authors any irregularity in the meandering pattern will give rise to a concentration of energy dissipation at that point with consequent acceleration of the processes which will tend to restore an even distribution of energy. The authors showed that a sine-generated curve was the curve of minimum total work and therefore the curve which permitted the most uniform consumption of energy per unit length of channel

13.17 What controls the geometrical properties of a meandering system ?

It is a matter of observation that the flatter the valley slope the finer the alluvium. The cause and effect are interrelated. The finer the material, the more readily it can be transported and therefore the lower the mean non-scour velocity and as a consequence the longer the channel course and greater the sinuosity of the river to reduce the velocity to the equilibrium condition.

Bank competence plays an important role in that the greater the sinuosity the smaller the radius of curvature and the greater the tractive force at the bends. Cohesive banks or banks protected by vegetation will permit greater sinuosity. Under these conditions the channel will be narrower and deeper than banks in loose material. In the latter case stability is achieved by a wider and therefore shallower channel which follows a less sinuous course. Reduction in tractive force and corresponding energy allocation is achieved by the longer wetted perimeter of the channel rather than the longer channel course.

14. RIVER CHANNEL RESPONSE TO MAN'S INTERVENTION

14.1 Channel shape

I have long held the view that the removal of vegetation along a river bank will produce the most dramatic change in channel geometry, and sediment transport (mainly the redistribution of sediment - see Technical Note No 70 and Technical Report No 87

I will not repeat the reasoning here.

14.2 Dam construction - reduction in sediment load

The major effect of dam construction is the large reduction in wash load and virtual total reduction in bed material load. The former will have little effect on downstream conditions but the latter will result in a gradual progressive reduction in the bed material downstream of the dam. Where rock bars control the river bed level the reduction in bed material should not significantly affect channel geometry or hydraulics.

14.3 Dam construction - change in flow regime

Prolonged releases of flows significantly higher than the natural low flow regime in the river could accelerate the erosion of unprotected banks. This would be the first and probably only major change that would occur.

14.4 Diversion of additional flows into small streams

Long term changes in channel geometry can be expected where large diversions into small streams take place, particularly in arid areas where the banks are poorly protected. However, it must be borne in mind that it is the river's natural high flows that are the major channel formative processes so changes can only be expected if the additional flows are in this range.

14.5 Enhanced soil erosion

I have previously expressed the view that as enhanced soil erosion will increase the wash load (and probably river flow as well), but not the bed material, it is unlikely to have a significant effect on river channel geometry or flow characteristics (see Technical Note 87 "The effect of river management practices on the sedimentation of estuaries").

14.6 Construction of embankments or other obstructions in a flood plain

The effect of these obstructions is to raise the upstream water level and so reduce channel slope. In the short term increased sediment deposition in the backwater zone can be expected until a new long term gradient has been established which will allow the sediment to be transported through the system once more.

15. REFERENCES

In this age of instant information retrieval systems, there seems little point in listing the dozens of papers, publications and text books that I consulted in preparing these notes.

The journals which published most of the papers were :-

Journal of the Hydraulics Division of the American Society of Civil Engineers.
Journal of Hydrology, Amsterdam.
Water Resources Research.
Proceedings of the Institution of Civil Engineers, London.
Sedimentology, Amsterdam.
Department of Water Affairs reports in the Professional Paper, Technical Report, and Technical Note series.

The authors who provided me with the most food for thought in chronological order of first publication referred to were :-

VANONI, V A
ALLEN, J R L
LEOPOLD, L B and LANGBEIN, W B
MADDOCK, THOMAS, Jr
ACKERS, P and CHARLTON, F G
YANG, C T
CHITALE, S V
RANA, S A , SIMONS, D B and MAHMOOD, K
ACKERS, P and WHITE, W R
QUICK, M C
SHEN, H W
PICKUP, G
HEY, R D
- and last but by no means least our own :-
ROOSEBOOM, A

Text books and publications which provided greater depth on the subject were :-

FLUVIAL PROCESSES IN GEOMORPHOLOGY. Leopold, Wolman and Miller. *Freeman and Company*, 1964 (This was not available when I wrote the notes but I had previously read through it).

HANDBOOK OF APPLIED HYDROLOGY. Edited by ven te Chow. *McGraw-Hill Book Company*. 1964

OPEN CHANNEL FLOW. F M Henderson. *Macmillan Company*. 1966

RIVER MECHANICS. Edited by Hsieh Wen Shen. *Colorado State University*. 1971

STOCHASTIC HYDRAULICS. Edited by Chao-Lin Chiu. *University of Pittsburgh Publications*. 1971

HYDRAULICS OF SEDIMENT TRANSPORT. Graf, W H. *McGraw-Hill Book Company*. 1971

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May you find as much interest in the subject as I did.

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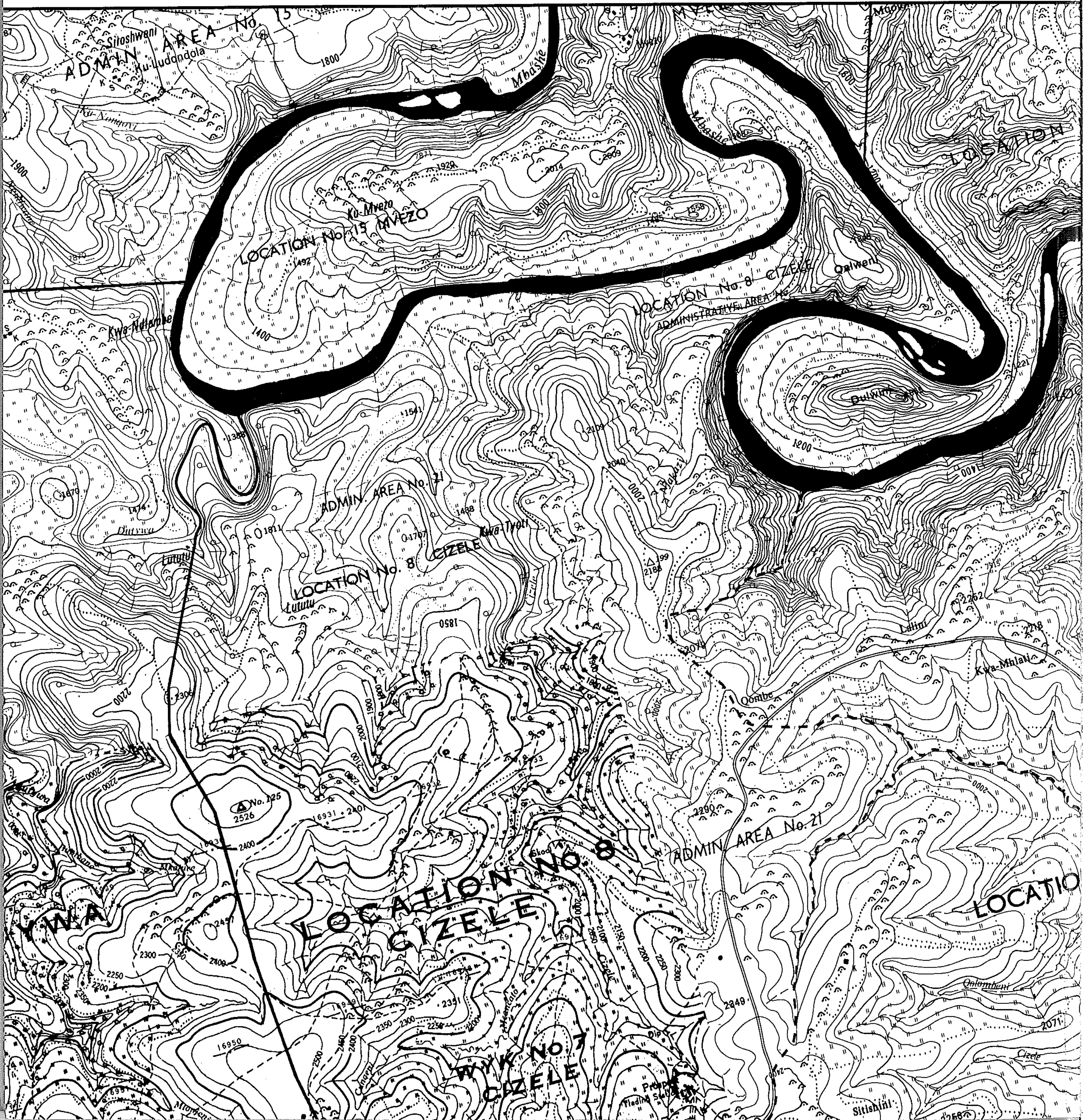
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Sitoshwani
Ku-Ludondola
Mbashe
LOCATION No. 15 MVEZO
Ku-Mvezo
Kwa-Ndlambe
LOCATION No. 8 CIZELE
ADMINISTRATIVE AREA No.
Qolweni
Dulwini
ADMIN. AREA No. 21
Kwa-Tyoti
LOCATION No. 8 CIZELE
Lututu
Dutywa
Qombe
Lalini
Kwa-Mhlati
ADMIN AREA No. 21
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LOCATIO
WYK No. 7 CIZELE
Qolombeni
Cizele
Sitishini
LOCATION